1. Consider the definite integral given by

$$\int_0^2 (4 + 2x^2)dx.$$

a. Divide the interval $x \in [0, 2]$ into $n = 2$ even subintervals, then apply both the midpoint rule and the trapezoid rule to estimate this integral.

b. Divide the interval $x \in [0, 2]$ into $n = 4$ even subintervals, then again apply both the midpoint rule and the trapezoid rule to estimate this integral.

c. It can be shown that the value of this integral is $\frac{40}{3}$. Give the percent error for each of the cases above, noting whether the estimate is high or low.

2. Consider the definite integral given by

$$\int_0^2 x^4 dx.$$

a. Divide the interval $x \in [0, 2]$ into $n = 4$ even subintervals, then apply the midpoint rule, the trapezoid rule, and Simpson's rule to estimate this integral.

b. It can be shown that the value of this integral is $\frac{32}{5}$. Give the percent error for each of the cases above, noting whether the estimate is high or low.

3. a. Find the domain and sketch the graph of the function

$$f(x) = \sqrt{4 - x}.$$

b. Use the midpoint rule and the trapezoid rule with $n = 4$ to estimate the area in the first quadrant ($x \geq 0$, $y \geq 0$) under this curve.

4. a. Sketch the graph of the function

$$f(x) = 8 + 2x - x^2.$$

Find all x and y-intercepts and the vertex.

b. Use the midpoint rule and the trapezoid rule with $n = 4$ to estimate the area in the first quadrant ($x \geq 0$, $y \geq 0$) under this curve.

c. Since this is a quadratic function, Simpson's rule gives the exact value. Use Simpson's rule with $n = 4$ to find the area in the first quadrant under this curve. Compare your numbers from the midpoint rule and the trapezoid rule to this answer. (Give the percent error of the approximations in Part b.)

5. Two researchers analyze seven years of population data for a particular animal that is given in the table below.

Year	0	1	2	3	4	5	6	7
Pop	12	18	27	32	28	17	12	21

a. Find the average population in the usual manner.

b. An alternate (often considered more accurate) method of computing the average population is given by the definite integral

$$P_{ave} = \frac{1}{7}\int_0^7 P(t)dt.$$

Use the trapezoid rule ($n = 7$) to estimate the average population from this definite integral and compare this answer to your answer in Part a.

6. Toxicity of a drug is often determined by the amount of drug in the blood times the length of time it remains at that level. This cumulative effect is found by integrating the amount of drug over the time that the dose is effective. Suppose that the amount of a drug $A(t)$ is measured over a period of time after taking a pill and its quantity is found to be

Hour	0	1	2	3	4	5	6	7	8	9	10
A	0.05	0.46	0.87	0.54	0.43	0.36	0.28	0.21	0.16	0.12	0.09

The cumulative dose is given by integral

$$\int_0^{10} A(t)\,dt.$$

Use the trapezoid rule ($n = 10$) to estimate the cumulative drug effect from the data above.

7. The trapezoid rule uses areas of trapezoids to approximate the area under a curve. (See diagram below.) Compute the area of the trapezoid in the diagram by computing the area of the rectangle of height $f(x_i)$ and the area of the triangle on top of the rectangle, then adding these areas together. With this information, explain how the formula for the Trapezoid rule combines these areas of trapezoids to give the formula in the lecture notes.