1. a.

$$\begin{aligned}\int\left(6\cos(3x)-\frac{2}{x^3}\right)dx &= 6\int\cos(3x)dx-2\int x^{-3}dx\\ &= 6\frac{\sin(3x)}{3}-2\frac{x^{-2}}{-2}+C=2\sin(3x)+\frac{1}{x^2}+C\end{aligned}$$

b.

$$\begin{aligned}\int\left(4x+e^{-3x}\right)dx &= 4\int x\,dx+\int e^{-3x}dx\\ &= 4\frac{x^2}{2}+\frac{e^{-3x}}{-3}+C=2x^2-\frac{1}{3}e^{-3x}+C\end{aligned}$$

c.

$$\begin{aligned}\int\left(4e^{-2x}+\frac{3}{\sqrt{x}}\right)dx &= -2e^{-2x}+3\int x^{-1/2}dx\\ &= -2e^{-2x}+6\sqrt{x}+C\end{aligned}$$

d.

$$\begin{aligned}\int\left(5x^2-1\right)^2dx &= \int\left(25\,x^4-10\,x^2+1\right)dx\\ &= 5\,x^5-\frac{10}{3}\,x^3+x+C\end{aligned}$$

e.

$$\begin{aligned}\int_0^{2\pi}(\cos(t/4)+t)\,dt &= \left(4\sin(t/4)+\frac{t^2}{2}\right)\Bigg|_{t=0}^{2\pi}\\ &= 4\sin(\pi/2)+\frac{4\pi^2}{2}-4\sin(0)-0=4+2\,\pi^2\end{aligned}$$

f.

$$\begin{aligned}\int_1^4\left(6x+\frac{2}{x}\right)dx &= \left(3x^2+2\ln|x|\right)\Big|_{x=1}^{4}\\ &= 48+2\,\ln(4)-3-2\,\ln(1)=45+4\,\ln(2)\end{aligned}$$

g.

$$\begin{aligned}\int_0^{\pi/2} (2 - 4\,\sin(2x))\,dx &= (2x + 2\,\cos(2x))\Big|_{x=0}^{\frac{\pi}{2}} \\ &= \pi + 2\,\cos(\pi) - 0 - 2\,\cos(0) = \pi - 4\end{aligned}$$

h.

$$\begin{aligned}\int_1^3 \left(x^2 - \frac{3}{x^2}\right)^2 dx &= \int_1^3 \left(x^4 - 6 + 9\,x^{-4}\right)^2 dx \\ &= \left(\frac{x^5}{5} - 6x - \frac{3}{x^3}\right)\Bigg|_{x=1}^{3} \\ &= \frac{243}{5} - 18 - \frac{1}{9} - \frac{1}{5} + 6 + 3 = \frac{1768}{45}\end{aligned}$$

2. a. This is a separable differential equation. It can be written

$$\int 2y\,dy = \int 3t^2\,dt \qquad \text{or} \qquad y^2 = t^3 + C.$$

It follows that $y(t) = \pm\sqrt{t^3 + C}$. The initial condition $y(0) = 4 = \sqrt{C}$, which implies $C = 16$. Hence, the solution is $y(t) = \sqrt{t^3 + 16}$.

b. This is a time varying differential equation. It can be written

$$y(t) = \int \left(2 - \frac{4}{t}\right) dt = 2\,t - 4\,\ln(t) + C.$$

The initial condition $y(1) = 5 = 2 + C$, which implies $C = 3$. Hence, the solution is $y(t) = 2\,t - 4\,\ln(t) + 3$.

c. This is a time varying differential equation. It can be written

$$y(t) = \int 4\cos(2t)dt = 2\,\sin(2\,t) + C.$$

The initial condition $y(0) = 3 = C$, which implies $C = 3$. Hence, the solution is $y(t) = 2\,\sin(2\,t) + 3$.

d. This is a separable differential equation. It can be written

$$\int \frac{dy}{y} = \int (2 - 0.2t)dt \qquad \text{or} \qquad \ln|y| = 2\,t - 0.1\,t^2 + C.$$

It follows that $y(t) = e^{2\,t - 0.1\,t^2 + C} = Ae^{2\,t - 0.1\,t^2}$. The initial condition $y(0) = 10 = A$, which implies $A = 10$. Hence, the solution is $y(t) = 10\,e^{2\,t - 0.1\,t^2}$.

e. This is a linear differential equation, so we first write $\frac{dy}{dt} = 2 + \frac{y}{3} = \frac{1}{3}(y+6)$. Thus, we make the substitution $z(t) = y(t) + 6$, giving the differential equation $\frac{dz}{dt} = \frac{1}{3}z$ with the initial condition $z(0) = y(0) + 6 = 8$. Thus, $z(t) = 8\,e^{t/3}$. It follows that $y(t) = 8\,e^{t/3} - 6$.

f. This is a separable differential equation. It can be written

$$\int e^y dy = \int e^t dt \qquad \text{or} \qquad e^y = e^t + C.$$

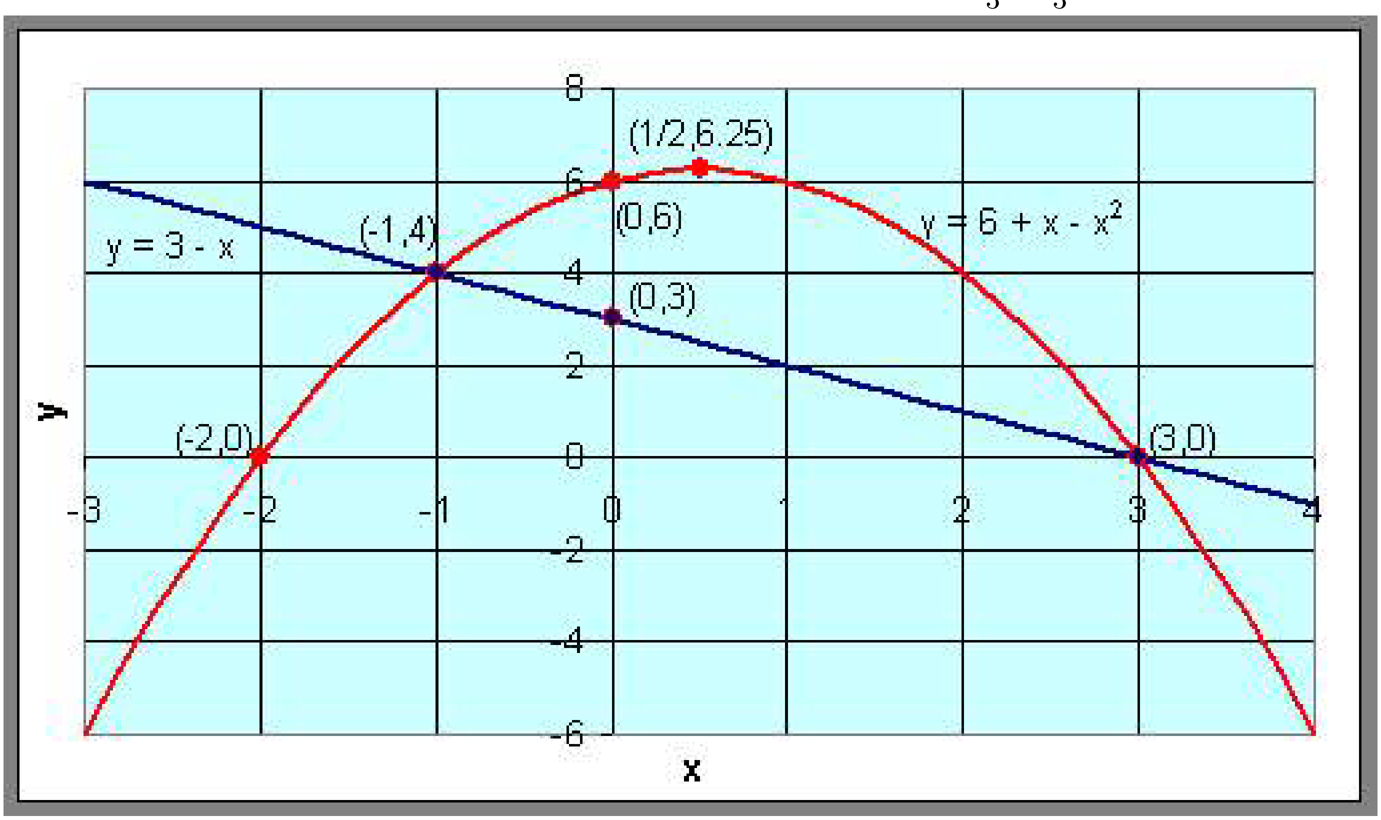
It follows that $y(t) = \ln(e^t + C)$. The initial condition $y(0) = 6 = \ln(1 + C)$, which implies $C = e^6 - 1$. Hence, the solution is $y(t) = \ln(e^t + e^6 - 1)$.

3. a. From the equation of the line, $y = 3 - x$, the x and y-intercepts are easily seen to be $(3, 0)$ and $(0, 3)$, respectively. Also, the slope of the line is $m = -1$. The equation for the parabola is $y = 6 + x - x^2 = -(x+2)(x-3)$. From this it is easy to see that the $x$intercepts are $(-2, 0)$ and $(3, 0)$, while the y-intercept is $(0, 6)$. The vertex of the parabola has its x-coordinate halfway between the x-intercepts, thus the vertex is $(\frac{1}{2}, 6\,\frac{1}{4})$.

Setting the two equations together, $3 - x = 6 + x - x^2$ or $x^2 - 2x - 3 = (x+1)(x-3) = 0$. Thus, the points of interesection are $(-1, 4)$ and $(3, 0)$. The graph can be seen below.

b. The area between the two curves is given by

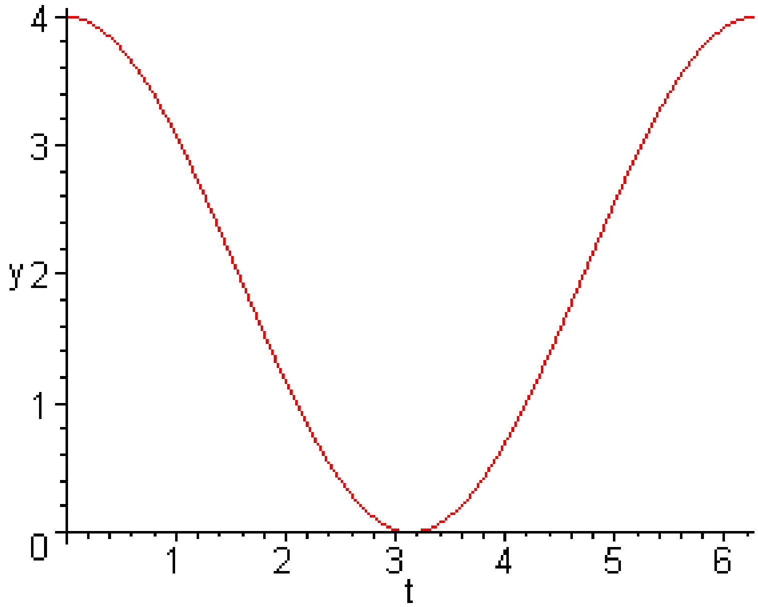
$$\begin{aligned} \int_{-1}^{3} \left(6 + x - x^2 - (3 - x)\right)\, dx &= \int_{-1}^{3} 3 + 2\,x - x^2 dx \\ &= \left(3x + x^2 - \frac{x^3}{3}\right)\Bigg|_{x=-1}^{3} \\ &= 9 + 9 - 9 + 3 - 1 - \frac{1}{3} = \frac{32}{3} \end{aligned}$$

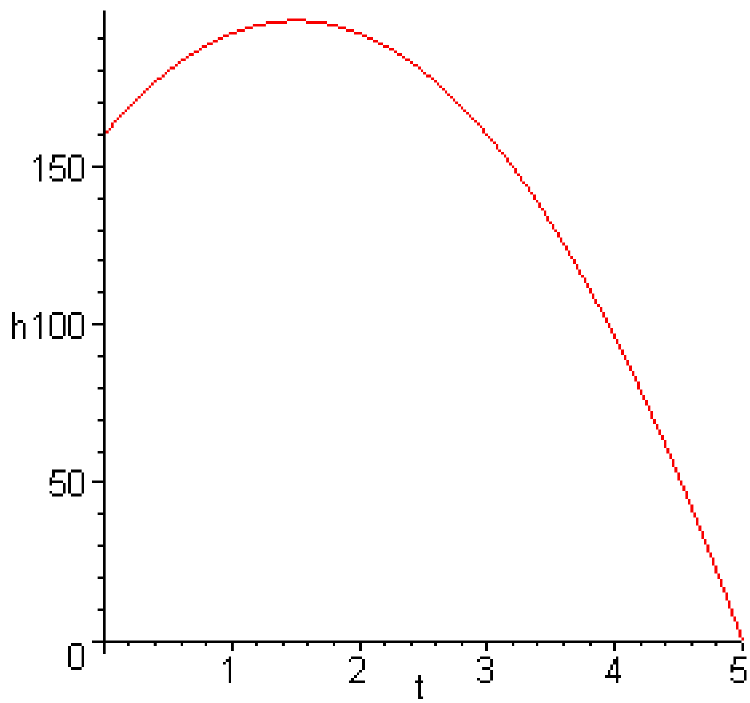


4. a. The period of this function is 2π. This is a typical cosine function with an amplitude of 2 and with the graph shifted up by 2 units. The graph can be seen below (left).

b. The area under the curve is given by

$$\begin{aligned} \int_0^{2\pi} 2\cos(t) + 2dt &= (2\sin(t) + 2t)\Big|_{x=0}^{2\pi} \\ &= 4\pi \end{aligned}$$





5. a. Since the acceleration of gravity is -32 ft/sec^2, the velocity of the ball is the integral, giving $v(t) = -32t + C$, which when combined with the initial condition $v(0) = 48$, gives $v(t) = 48 - 32t$. The velocity is integrated to give the height of the ball

$$h(t) = \int v(t)\,dt = \int (-32t + 48)dt = -16t^2 + 48t + C.$$

With the initial height, $h(0) = 160$, so $h(t) = -16t^2 + 48t + 160$. The maximum occurs when $v(t) = 0$, so $t = 3/2$ sec. It follows that the maximum height of the ball is $h(3/2) = 196$ ft.

b. The ball hits the ground when $h(t) = -16(t^2 - 3t - 10) = -16(t + 2)(t - 5) = 0$, so at $t = 5$ sec. The velocity is $v(5) = -112$ ft/sec.

c. The graph for the height of the ball is shown above (right) for $t \geq 0$.

6. Integrating the acceleration due to gravity as in the previous problem, we see that the velocity is given by $v(t) = v_0 - 32\,t$. Similarly, the height is the integral of the velocity (as above), so $h(t) = \int (v_0 - 32\,t)dt = -16\,t^2 + v_0 t$, where the integration constant is zero, since the initial height is zero. The maximum height occurs when the velocity is zero, so $t = v_0/32$. But

$$h(v_0/32) = \frac{v_0^2}{32} - \frac{v_0^2}{64} = \frac{v_0^2}{64} = 8.$$

It follows that $v_0^2 = 512$ or $v_0 = 16\sqrt{2}$, which is the initial upward velocity. The length of time that the kangeroo stays in the air is twice the length of time to reach the maximum, so it stays in the air for $t = \sqrt{2}$ sec.

7. a. The solution to the Malthusian growth model is given by $P(t) = 100\,e^{0.2\,t}$. This population doubles when $100\,e^{0.2\,t} = 200$ or $e^{0.2\,t} = 2$, so $t = 5\ln(2) \simeq 3.466$ yrs.

b. This model, including the modification for habitat encroachment, is a separable differential equation. It can be written

$$\int \frac{dP}{P} = \int (0.2 - 0.02t)dt \qquad \text{or} \qquad \ln|P| = 0.2\,t - 0.01\,t^2 + C.$$

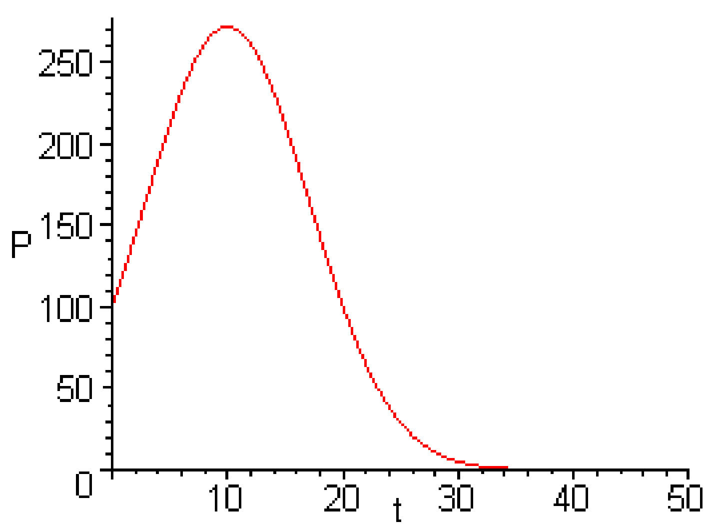
It follows that $P(t) = e^{0.2\,t - 0.01\,t^2 + C} = Ae^{0.2\,t-0.01\,t^2}$. The initial condition $P(0) = 100 = A$, which implies $A = 100$. Hence, the solution satisfies

$$P(t) = 100\,e^{0.2\,t-0.01\,t^2}.$$

c. We find the maximum by differentiating and setting it equal to zero,

$$P'(t) = 100\,e^{0.2\,t-0.01\,t^2}(0.2 - 0.02t) = 0.$$

So $0.2 - 0.02t = 0$, which implies that $t = 10$. Thus, the maximum of population is $P(10) = 100\,e \simeq 271.8$. If we solve $P(t) = 100\,e^{0.2\,t-0.01\,t^2} = 100$, then this is equivalent to $e^{0.2\,t-0.01\,t^2} = 1$ or $0.2\,t - 0.01\,t^2 = -0.01\,t(t-20) = 0$. Thus, either $t = 20$ (or 0), so the population returns to 100 after 20 years. The graph of the population can be seen below.



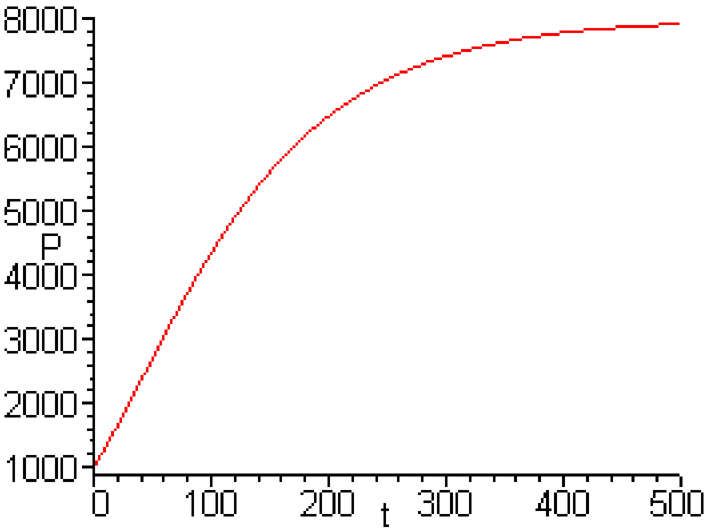
8. a. This population of cells in a declining medium satisfies a separable differential equation, which can be written

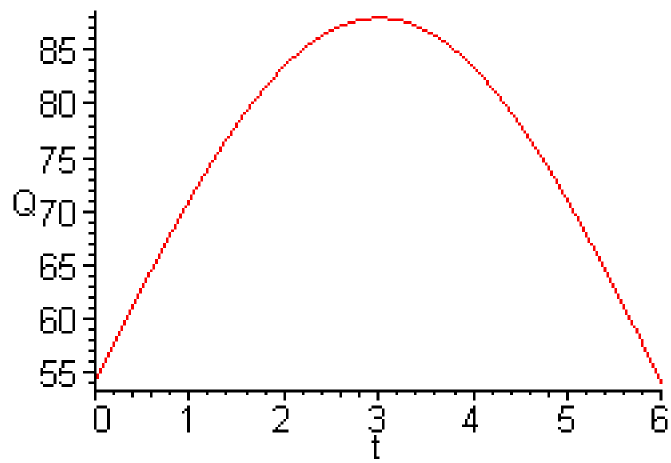
$$\int P^{-2/3}dP = \int 0.3\,e^{-0.01t}dt \qquad \text{or} \qquad 3\,P^{1/3}(t) = -30\,e^{-0.01t} + 3C.$$

It follows that $P^{1/3}(t) = -10\,e^{-0.01t} + C$, so $P(t) = \left(C - 10\,e^{-0.01t}\right)^3$. The initial condition $P(0) = 1000 = (C-10)^3$, which implies $C = 20$. The solution is given by

$$P(t) = \left(20 - 10e^{-0.01\,t}\right)^3.$$

b. This population doubles when $P(t) = \left(20 - 10e^{-0.01\,t}\right)^3 = 2000$, so $20 - 10e^{-0.01\,t} = 10\sqrt[3]{2}$ or $e^{-0.01\,t} = 2 - \sqrt[3]{2}$. It follows that $t = 100\,\ln\left(\frac{1}{2-\sqrt[3]{2}}\right) \simeq 30.1$ hr. For large t, $\lim_{t\to\infty} e^{-0.01\,t} = 0$, so $\lim_{t\to\infty} P(t) = 20^3 = 8000$. Thus, there is a horizontal asymptote at $P = 8000$, so the population tends towards this value. The graph of the population can be seen below.





9. a. The maximum population for $P(t) = 54 + 24\,t - 4\,t^2$ is found by differentiating with $P'(t) = 24 - 8\,t$, which is zero at $t = 3$. This gives a maximum population of $P(3) = 90$.

b. The maximum population for $Q(t) = 54 + 34\sin\left(\frac{\pi}{6}t\right)$ occurs when $t = 3$, which is when the argument of the sine function is at $\pi/2$. This gives a maximum of $Q(3) = 88$. A graph of the function is above.

c. The average of the data is $\frac{54+73+85+89+86+75+53}{7} = 73.57$. The averages from the integrals are

$$\begin{aligned} P_{ave} = \frac{1}{6}\int_0^6 P(t)dt &= \frac{1}{6}\int_0^6 54 + 24\,t - 4\,t^2 dt \\ &= \frac{1}{6}\left(54\,t + 12\,t^2 - \frac{4}{3}t^3\right)\Big|_{x=0}^{6} \\ &= \frac{1}{6}(54(6) + 72(6) - 48(6)) = 78 \end{aligned}$$

and

$$
\begin{aligned}
Q_{ave} = \frac{1}{6}\int_0^6 Q(t)dt &= \frac{1}{6}\int_0^6 54 + 34\,\sin(1/6\,\pi\,t)dt \\
&= \frac{1}{6}\left(54\,t - \frac{204}{\pi}\,\cos(1/6\,\pi\,t)\right)\Big|_{x=0}^{6} \\
&= \frac{1}{6}\left(54(6) - \frac{204}{\pi}(\cos(\pi) - \cos(0))\right) = 54 + \frac{68}{\pi} = 75.65.
\end{aligned}
$$

10. a. The equation for the weight of the swordfish is a linear differential equation, so we first write $\frac{dw}{dt} = 0.015(1000 - w) = -0.015(w - 1000)$. We make the substitution $z(t) = w(t) - 1000$, giving the differential equation $\frac{dz}{dt} = -0.015z$ with the initial condition $z(0) = w(0) - 1000 = -1000$. Thus, $z(t) = -1000\,e^{-0.015\,t}$. It follows that $w(t) = 1000 - 1000\,e^{-0.015\,t}$. The swordfish reaches 70 kg when $1000 - 1000\,e^{-0.015\,t} = 70$ or $e^{0.015\,t} = \frac{1000}{930}$. Thus, it takes $t = \frac{200}{3}\ln\left(\frac{100}{93}\right) \simeq 4.838$ yrs to reach maturity.

b. The mercury (Hg) accumulates in swordfish according to the differential equation, which is a time varying equation. It follows that upon integration that

$$
\begin{aligned}
H(t) &= 0.01\int(1000 - 1000\,e^{-0.015\,t})dt \\
&= 10t - \frac{2000}{3}\,e^{-0.015\,t} + C
\end{aligned}
$$

With the initial condition $H(0) = 0$, the solution becomes

$$H(t) = 10t - \frac{2000}{3}\,e^{-0.015\,t} + \frac{2000}{3}.$$

From this equation, it follows that $H(3) = 59.3$ and $H(20) = 372.8$ mg of Hg.

c. The formula for the concentration of Hg, $c(t)$ (in μg/g) in swordfish satisfies

$$c(t) = H(t)/w(t) = \frac{10t - \frac{2000}{3}\,e^{-0.015\,t} + \frac{2000}{3}}{1000 - 1000\,e^{-0.015\,t}}.$$

It follows that $c(3) = 1.35$ and $c(20) = 1.44\ \ \mu$g/g.