

1. a. $f'(x) = 3 \cos(3x - 5) - \frac{3 \sin(3x)}{\cos(3x)}$.

b. Rewrite the function as $g(x) = 4 (\cos(x^2 + 2))^{-1} - (x^2 - \sin^3(x^2))^4$, then the chain rule gives

$$g'(x) = -8x (\cos(x^2 + 2))^{-2} \sin(x^2 + 2) - 4(x^2 - \sin^3(x^2))^3 (2x - 6x \sin^2(x^2) \cos(x^2)).$$

c. Use the quotient rule and product rule

$$h'(x) = \frac{(x^3 + \cos(4x))(4x^3 - 2e^{-2x}) - (x^4 + e^{-2x})(3x^2 - 4 \sin(4x))}{(x^3 + \cos(4x))^2} - e^{-x} \cos(2x) - 2e^{-x} \sin(2x).$$

d. With the product rule and chain rule,

$$k'(x) = -3x^2(x^2 - 5)^3 \sin(x^3) + 6x(x^2 - 5)^2 \cos(x^3) - 2 \cos(2x)e^{\sin(2x)}.$$

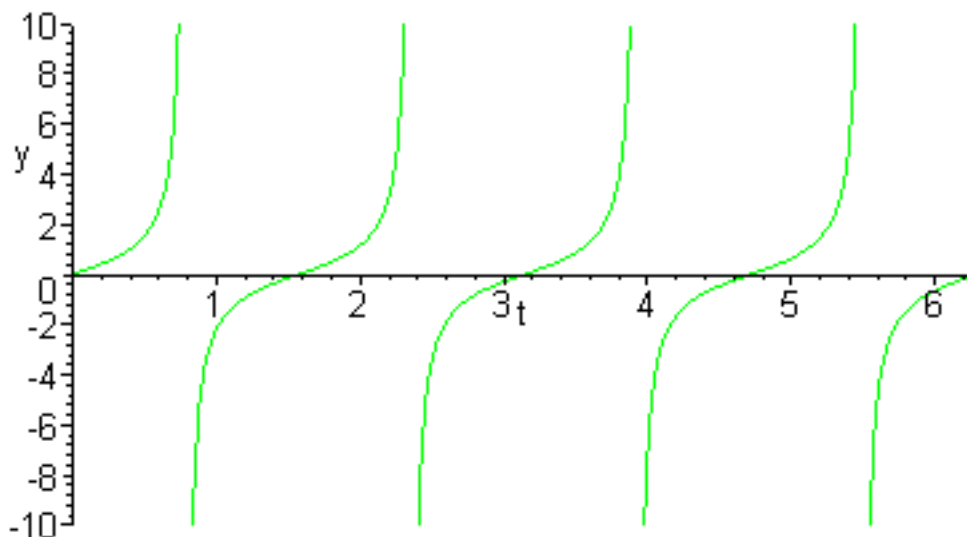
2. a. The derivative is given by

$$f'(t) = \frac{2 \cos(2t) \cos(2t) + 2 \sin(2t) \sin(2t)}{\cos^2(2t)} = \frac{2}{\cos^2(2t)},$$

since $\sin^2(2t) + \cos^2(2t) = 1$. It follows that $f'(0) = 2/\cos^2(0) = 2$. Notice that since the denominator is squared, it follows that the derivative is always positive for all t that the derivative is defined.

b. $f(t)$ is zero when $\sin(2t) = 0$. The sine function is zero when its argument is an integer multiple of π . For $t \in [0, 2\pi]$, $f(t) = 0$ at $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$. The cosine function is zero when its argument is $\pi/2 + n\pi$ for n an integer. Thus, the vertical asymptotes occur halfway between zeroes of f , so at $t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

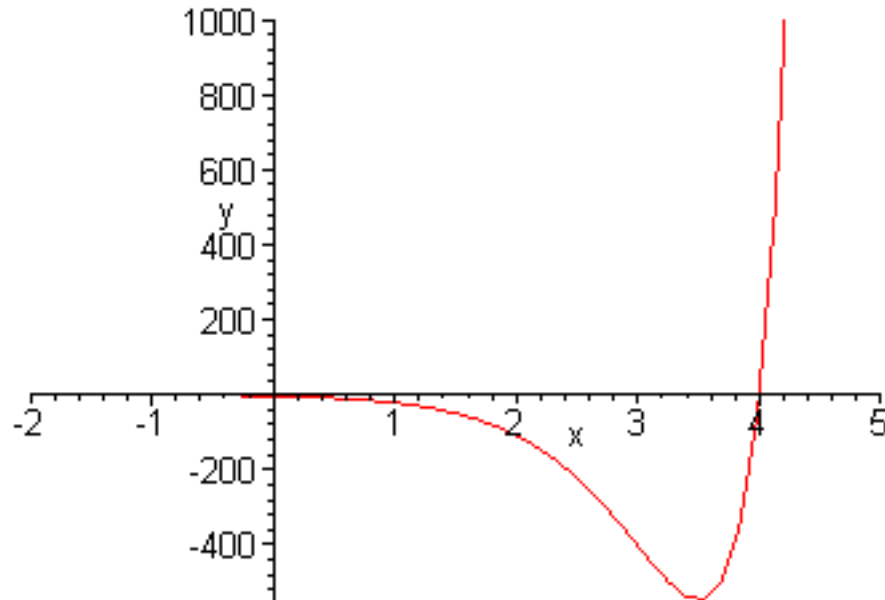
c. The graph of $f(t)$ for $t \in [0, 2\pi]$ is below.



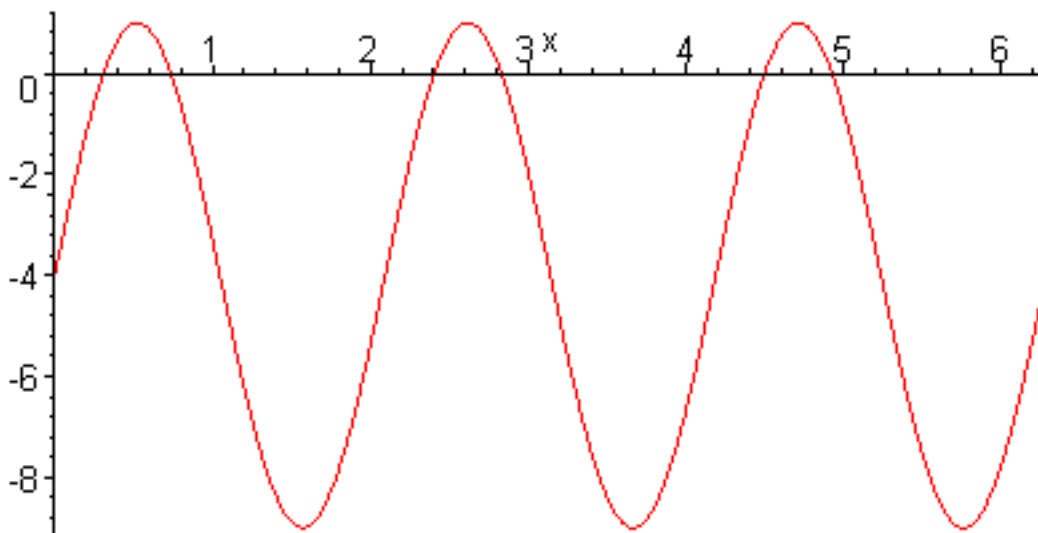
3. The y -intercept satisfies $f(0) = -4$. The x -intercept satisfies $f(x) = 0$, which gives $x = 4$. The derivative satisfies

$$f'(x) = 2(x - 4)e^{2x} + e^{2x} = (2x - 7)e^{2x}.$$

The derivative is zero at $x = 3.5$, so $f(3.5) = -0.5e^7 \simeq -548.3$. Thus, a minimum occurs at $(3.5, -0.5e^7) \simeq (3.5, -548.3)$. There is a horizontal asymptote with $y = 0$ as $x \rightarrow -\infty$. The graph is below.



4. The function, $y = 5 \sin(3x) - 4$, has a period of $x = 2\pi/3$. The function oscillates about $y = -4$ with an amplitude of 5. It begins at $(0, -4)$, goes to a maximum at $(\pi/6, 1)$, continues through $(\pi/3, -4)$, then reaches a minimum at $(\pi/2, -9)$, and ends its cycle at $(2\pi/3, -4)$. The maxima occur at $x = \pi/6, 5\pi/6, 3\pi/2$. The graph of the function is below.



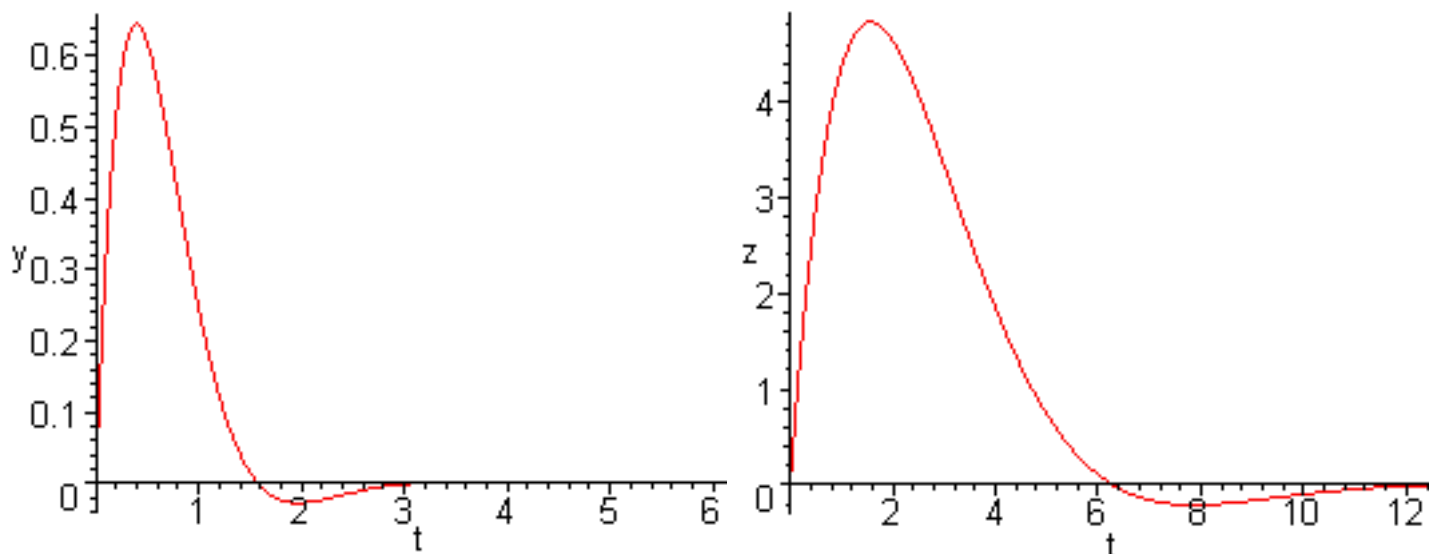
5. a. The damped spring-mass system, $y(t) = 2e^{-2t} \sin(2t)$, has a velocity

$$\begin{aligned} v(t) = y'(t) &= 4e^{-2t} \cos(2t) - 4e^{-2t} \sin(2t) \\ &= 4e^{-2t}(\cos(2t) - \sin(2t)) \end{aligned}$$

b. The maximum occurs when $\cos(2t) = \sin(2t)$ or $t = \pi/8$. Thus, the maximum is

$$y(\pi/8) = 2e^{-\pi/4} \sin(\pi/4) \simeq 0.6448.$$

The mass returns to $y(t) = 0$ when $\sin(2t) = \sin(\pi)$ or $t = \pi/2$. Below to the left is a graph of the mass.



6. a. The basilar fiber vibrates through zero when the argument of $\sin(t/2)$ equals $n\pi$ for n an integer. It follows that the zeroes occur when $t = 0, 2\pi, 4\pi$.

b. The velocity is given by

$$\begin{aligned} v(t) = z'(t) &= \frac{15}{2}e^{-t/2} \cos(t/2) - \frac{15}{2}e^{-t/2} \sin(t/2) \\ &= \frac{15}{2}e^{-t/2} (\cos(t/2) - \sin(t/2)) \end{aligned}$$

c. The extrema occur when $\cos(t/2) = \sin(t/2)$, so $t/2 = \pi/4 + n\pi$ for n an integer. There is a maximum at $t = \pi/2$ with

$$z(\pi/2) = 15e^{-\pi/4} \sin(\pi/4) \simeq 4.836.$$

This is followed by a minimum at $t = 5\pi/2$ with

$$z(5\pi/2) = 15e^{-5\pi/4} \sin(5\pi/4) \simeq -0.2090.$$

The graph of $z(t)$ for $t \in [0, 4\pi]$ is shown above to the right.

7. a. The periodic contractions of 10/min implies that $0.1\omega = 2\pi$ or $\omega = 20\pi$. The average value $A = \frac{4+1}{2} = 2.5$, while the amplitude is given by $B = 4 - 2.5 = 1.5$. Thus, the radius of the small intestine is given by

$$R(t) = 2.5 + 1.5 \cos(20\pi t).$$

b. The graph of $R(t)$ for $t \in [0, 0.2]$ is shown below to the left. The maxima occur at $t = 0, 0.1, 0.2$ min, and the minima are halfway between the maxima with $t = 0.05, 0.15$ min.

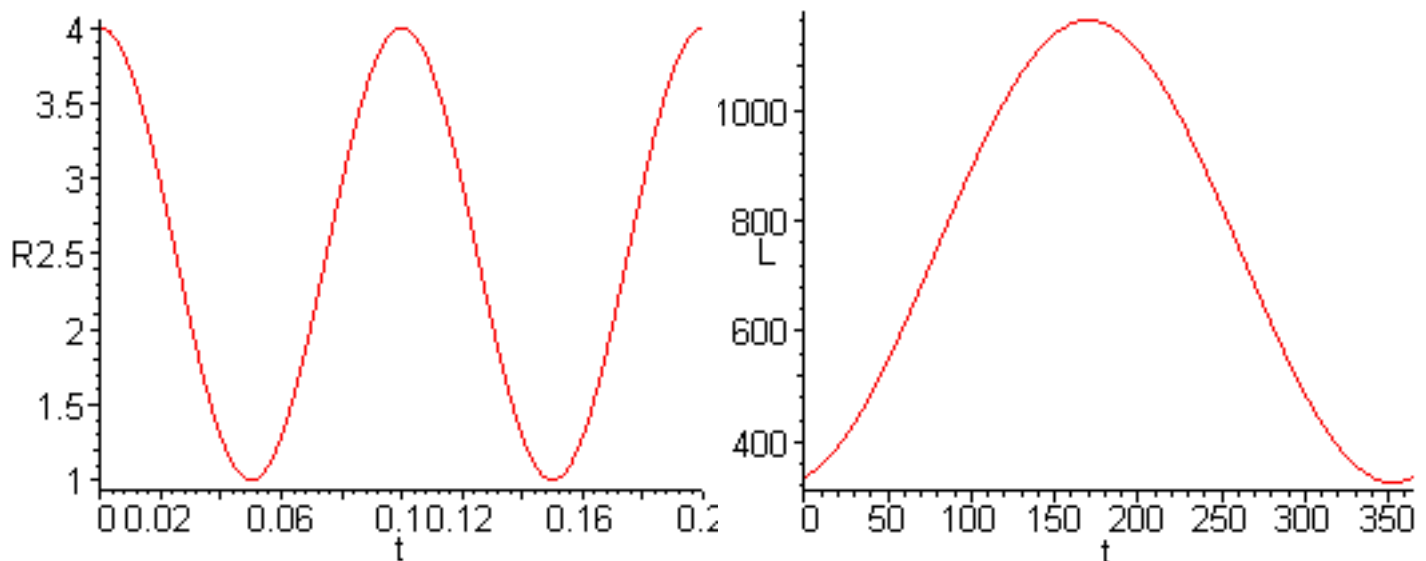
c. The derivative of $R(t)$ is given by

$$R'(t) = -30\pi \sin(20\pi t).$$

The maximum rate of decrease is when the sine function is 1, which occurs when $20\pi t = \pi/2$ or $t = \frac{1}{40} = 0.025$ min with

$$R'(0.025) = -30\pi \simeq 94.25 \text{ cm/min.}$$

and find the maximum rate of decrease in the radius $R(t)$ (in cm/min) and the first time after $t = 0$ when this occurs.



8. a. The period is 365 days, so $365\omega = 2\pi$ or $\omega = \frac{2\pi}{365} \simeq 0.01721$. The average length of time is $\alpha = \frac{1162+327}{2} = 744.5$ min. The amplitude is given by $\beta = 1162 - 744.5 = 417.5$ min. The maximum occurs on day 170, so $\omega(170 - \phi) = \pi/2$ (based on the maximum of the sine function). Thus, $170 - \phi = \frac{365}{4} = 91.25$ or $\phi = 78.75$ day. It follows that

$$L(t) = 744.5 + 417.5 \sin(0.01721(t - 78.75)).$$

The length of day for Ground Hog's day is $L(32) = 744.5 + 417.5 \sin(0.01721(32 - 78.75)) = 443.7$ min in Anchorage.

b. The derivative of $L'(t) = 7.185 \cos(0.01721(t - 78.75))$. The maximum rate of change occurs when cosine is 1, so $L'(78.75) = 7.185$ min/day, which occurs on day 78.75 or about March 21, the first day of spring. A graph is shown above to the right.

9. The area of the brochure is $A = xy = 125$, where x is the width of the page and y is the length of the page. The area of the printed page, which is to be maximized is given by

$$P = (x - 4)(y - 5).$$

From the constraint on the page area, we have $y = 125/x$, which when substituted above gives

$$P(x) = (x - 4) \left(\frac{125}{x} - 5 \right) = 125 - \frac{500}{x} - 5x + 20 = 145 - 500x^{-1} - 5x.$$

The maximum is found by differentiation, which gives

$$P'(x) = 500x^{-2} - 5 = \frac{5(100 - x^2)}{x^2}.$$

This is zero when $x = 10$. It follows that $y = 12.5$. So the brochure has the dimensions 10×12.5 with the printed region having dimensions 6×7.5 or 45 in^2 .

10. Combining the number of drops with the energy function, we have

$$E(h) = hN(h) = h \left(1 + \frac{10}{h-1} \right) = h \left(\frac{h-1+10}{h-1} \right) = \frac{h^2 + 9h}{h-1}.$$

This is differentiated to give

$$E'(h) = \frac{(h-1)(2h+9) - (h^2+9h)}{(h-1)^2} = \frac{h^2 - 2h - 9}{(h-1)^2}.$$

A minimum occurs when $h^2 - 2h - 9 = 0$, so

$$h = 1 \pm \sqrt{10} = -2.1623, 4.1623.$$

It follows that the minimum energy occurs when $h = 1 + \sqrt{10} = 4.1623 \text{ m}$, which give the height that a crow should fly to minimize the energy needed to break open a walnut.

11. From the diagrams, we have that $r^2 + h^2 = a^2$, which gives $h^2 = a^2 - r^2$. The circumference of the base of the cone is $2\pi r = a\theta$, where θ is in radians. (Radians are an easy means of determining the length of a sector of a circle.) Thus, $r = a\theta/2\pi$. It follows that $h^2 = a^2 - a^2\theta^2/(4\pi^2)$. The volume of the water cup is given by

$$\begin{aligned} V &= \frac{\pi r^2 h}{3} = \frac{\pi}{3} \left(\frac{a\theta}{2\pi} \right)^2 \sqrt{a^2 - \frac{a^2\theta^2}{4\pi^2}} \\ V(\theta) &= \frac{a^3\theta^2}{12\pi} \sqrt{1 - \frac{\theta^2}{4\pi^2}} = \frac{a^3}{12\pi} \theta^2 \left(1 - \frac{\theta^2}{4\pi^2} \right)^{1/2}. \end{aligned}$$

This expression is differentiated with respect to θ .

$$\begin{aligned}
 V'(\theta) &= \frac{a^3}{12\pi} \left(\frac{\theta^2}{2} \left(1 - \frac{\theta^2}{4\pi^2} \right)^{-1/2} \left(\frac{-2\theta}{4\pi^2} \right) + 2\theta \left(1 - \frac{\theta^2}{4\pi^2} \right)^{1/2} \right) \\
 &= \frac{a^3}{12\pi} \left(1 - \frac{\theta^2}{4\pi^2} \right)^{1/2} \left(\frac{-\theta^3}{4\pi^2} + 2\theta \left(1 - \frac{\theta^2}{4\pi^2} \right) \right) \\
 &= \frac{a^3\theta}{12\pi} \left(1 - \frac{\theta^2}{4\pi^2} \right)^{1/2} \left(2 - \frac{3\theta^2}{4\pi^2} \right)
 \end{aligned}$$

The maximum is found by setting this derivative above equal to zero, so $2 - \frac{3\theta^2}{4\pi^2} = 0$. It follows that $\theta^2 = 8\pi^2/3$ or

$$\theta = 2\pi\sqrt{\frac{2}{3}} \simeq 5.1302.$$

Thus, $\theta = 2\pi\sqrt{\frac{2}{3}} \simeq 5.1302$ radians (which is about 294°), so a sector of 1.1530 radians or about 66° is removed. The dimensions of the cone should have a radius of $r = a\sqrt{\frac{2}{3}} \simeq 0.8165a$ and a height of $h = a\sqrt{\frac{1}{3}} \simeq 0.57735a$.

12. a. The first two iterations are

$$B_1 = B_0 + 0.03B_0 \left(1 - \frac{B_0}{500,000} \right) = 10000 + 300 \left(1 - \frac{10,000}{500,000} \right) = 10,294$$

$$B_2 = B_1 + 0.03B_1 \left(1 - \frac{B_1}{500,000} \right) = 10,294 + 0.03(10,294) \left(1 - \frac{10,294}{500,000} \right) = 10596.5$$

b. The equilibria satisfy

$$B_e = B_e + 0.03B_e \left(1 - \frac{B_e}{500,000} \right) \quad \text{or} \quad 0.03B_e \left(1 - \frac{B_e}{500,000} \right) = 0.$$

From this equation, it follows that either $B_e = 0$ or $1 - \frac{B_e}{500,000} = 0$, which gives $B_e = 500,000$. Thus, the equilibria are $B_e = 0$ and $500,000$.

The updating function is given by

$$F(B_n) = B_n + 0.03B_n \left(1 - \frac{B_n}{500,000} \right) = 1.03B_n - \frac{0.03}{500,000}B_n^2.$$

Its derivative is

$$F'(B_n) = 1.03 - \frac{0.06}{500,000}B_n.$$

At $B_e = 0$, $F'(0) = 1.03 > 1$, so the equilibrium at $B_e = 0$ is unstable and solutions growing monotonically away from this equilibrium. At $B_e = 500,000$, $F'(500,000) = 0.97 < 1$, so the equilibrium at $B_e = 500,000$ is stable and solutions growing monotonically toward this equilibrium.