1. Evaluate the following integrals:

a. $\int \left(6\cos(3x) - \frac{2}{x^3}\right) dx$,

b. $\int \left(4x + e^{-3x}\right) dx$.

c. $\int \left(4e^{-2x} + \frac{3}{\sqrt{x}}\right) dx$,

d. $\int \left(5x^2 - 1\right)^2 dx$.

e. $\int_0^{2\pi} (\cos(t/4) + t)\, dt$,

f. $\int_1^4 \left(6x + \frac{2}{x}\right) dx$.

g. $\int_0^{\pi/2} (2 - 4\sin(2x))\, dx$,

h. $\int_1^3 \left(x^2 - \frac{3}{x^2}\right)^2 dx$.

2. Solve the following initial value problems:

a. $\frac{dy}{dt} = \frac{3t^2}{2y}, \quad y(0) = 4$,

b. $\frac{dy}{dt} = 2 - \frac{4}{t}, \quad y(1) = 5$,

c. $\frac{dy}{dt} = 4\cos(2t), \quad y(0) = 3$,

d. $\frac{dy}{dt} = (2 - 0.2t)y, \quad y(0) = 10$,

e. $\frac{dy}{dt} = 2 + \frac{y}{3}, \quad y(0) = 2$,

f. $\frac{dy}{dt} = e^{t-y}, \quad y(0) = 6$,

3. Consider the curves $y = 3 - x$ and $y = 6 + x - x^2$.

a. Find all the x and y-intercepts for both curves. Determine the slope of the line and the vertex of the parabola. Find the points of intersection, then sketch the graph of these curves. (Label the points clearly.)

b. Set up and solve the integral that determines the area between the two curves.

4. Consider the function $y = 2\cos(t) + 2$.

a. Find the period of this function and sketch a graph for $t \in [0, 2\pi]$.

b. Find the area under the curve in Part a.

5. A ball is tossed vertically into the air (with only gravity acting on it) with an initial velocity of 48 ft/sec from a 160 ft platform ($h'(0) = v(0) = 48$ and $h(0) = 160$). Assume that its height, $h(t)$ (in ft), above the ground t seconds after it is thrown satisfies the differential equation $h'' = -32$ ft/sec^2.

a. What is the maximum height of the ball and when does it occur?

b. When does the ball hit the ground and what is its velocity then?

c. Sketch a graph of the height of the ball vs. t for $t \geq 0$.

6. A kangeroo can leap vertically 8 ft. Determine equations describing the velocity and height of the kangeroo as functions of time using this assumption on the maximum height it can achieve, that is find $v(t)$ and $h(t)$ including numerical values of all constants in these formulae. How long is the kangeroo in the air and what is the animal's initial upward velocity? (Use the acceleration due to gravity as 32 ft/sec^2.)

7. a. Consider the Malthusian growth model for a particular animal that has recently colonized some region

$$P' = 0.2P, \qquad P(0) = 100,$$

where t is in years. Solve this differential equation and determine how long it takes for this population to double.

b. Because of habitat encroachment, this animal is losing its range for expansion. This results in a growth rate that is time dependent. Suppose that the population satisfies the modified Malthusian growth model

$$P' = (0.2 - 0.02t)P, \qquad P(0) = 100.$$

Solve this differential equation.

c. Find the maximum of this population and what year this occurs. Also, determine when the population returns to 100. Sketch a graph for this population.

8. a. Consider the growth of a population of cells in a declining medium. If the population growth depends on the absorption of the medium through the cell surface and the medium is decaying exponentially, then a differential equation for this population is given by

$$\frac{dP}{dt} = 0.3e^{-0.01t}P^{2/3}, \qquad P(0) = 1000,$$

where the initial population is 1000 and t is in hours. Solve this differential equation.

b. Find how long it takes for this population to double. What happens to this population for very large time (*i.e.*, find any horizontal asymptotes)? Sketch a graph for this population.

9. Two researchers analyze six years of population data for a particular animal that is given in the table below.

Year	0	1	2	3	4	5	6
Pop	54	73	85	89	86	75	53

a. The first researcher fits the data with the quadratic equation

$$P(t) = 54 + 24t - 4t^2.$$

Find the maximum population using this approximation to the data and when it occurs. (Show how you compute this maximum.)

b. The second researcher fits the data with the curve

$$Q(t) = 54 + 34\sin\left(\frac{\pi}{6}t\right).$$

Sketch a graph of this curve and give where the maximum population occurs.

c. Find the average populations using Parts a. and b. by computing the definite integrals

$$P_{ave} = \frac{1}{6}\int_0^6 P(t)dt \qquad \text{and} \qquad Q_{ave} = \frac{1}{6}\int_0^6 Q(t)dt.$$

10. There is a tremendous controversy about swordish. It is a very popular sportfish and top ocean predator. There is a boycott of this type of fish because the average harvest size has dropped well below the size of sexual maturity. It is also one of the earliest fish that was recognized to have a buildup of mercury. Sadly, the current swordfish that are being served are too small and young to even have a chance to buildup toxic levels of mercury with the average catch size being only 40 kg, which is less than 3 years old.

a. Swordfish can get extremely large, exceeding 1000 kg. However, it grows slowly (and if not overfished can live a long time), so the weight of a swordfish satisfies the following differential equation:

$$\frac{dw}{dt} = 0.015(1000 - w), \qquad w(0) = 0,$$

where w is the weight in kg and t is the time in years. Solve this differential equation. Use the results to determine how long it takes to produce a mature 70 kg swordfish.

b. The mercury (Hg) accumulates as the swordfish grows and is not removed. Assume that the intake of Hg is proportional to the weight of the swordfish, so satisfies the differential equation

$$\frac{dH}{dt} = kw(t), \qquad H(0) = 0,$$

with $k = 0.01$ (mg of Hg/kg-yr) and H being the mg of Hg in a swordfish. Solve this differential equation. Find the amount of Hg in swordfish that are 3 and 20 years old.

c. If the Hg is uniformly spread in the swordfish, then the concentration of Hg, $c(t)$ (in μg/g), would be given by the formula

$$c(t) = H(t)/w(t).$$

Find the weight of swordfish, $w(t)$, and concentration of Hg, $c(t)$, at times $t = 3$ and 20 years. (Note that officials get concerned when Hg level reaches 0.1 μg/g.)