1. Differentiate the following: (Do **NOT** simplify!)

a. $f(x) = \sin(3x - 5) + \ln(\cos(3x))$

b. $g(x) = \dfrac{4}{\cos(x^2 + 2)} - (x^2 - \sin^3(x^2))^4$

c. $h(x) = \dfrac{x^4 + e^{-2x}}{x^3 + \cos(4x)} + e^{-x}\cos(2x)$

d. $k(x) = (x^2 - 5)^3 \cos(x^3) - e^{\sin(2x)}$

2. Consider the function:

$$f(t) = \frac{\sin(2t)}{\cos(2t)}.$$

a. Find the derivative of $f(t)$. Evaluate $f'(0)$. Is the derivative positive, negative, or both for $t \geq 0$?

b. For $t \in [0, 2\pi]$, find all zeroes of $f(t)$. Also, determine where the function is undefined (zeroes of the denominator). The zeroes in the denominator give the location of the vertical asymptotes.

c. Use the information from Parts a and b to sketch a graph of $f(t)$ for $t \in [0, 2\pi]$.

3. Consider the function $f(x) = (x - 4)e^{2x}$. Find the x- and y-intercepts and all minima and maxima. Find any asymptotes, then sketch the graph.

4. Consider the trigonometric function:

$$y = 5\sin(3x) - 4, \qquad x \in [0, 2\pi].$$

Find the period of this function. Give the x and y values for all maxima in the specified interval. Sketch the graph of this function.

5. A damped spring-mass system has a solution of the form

$$y(t) = 2e^{-2t}\sin(2t),$$

where $y(t)$ measures the distance in centimeters from the equilibrium position and t is in seconds.

a. Find the velocity of the mass by computing the derivative, $v(t) = y'(t)$.

b. Find the time $t > 0$ and the position y when the mass is at a maximum. Also, determine the first time after $t = 0$ when $y(t) = 0$ again.

6. The displacement of specific fibers on the basilar membrane stimulates the hair cells, which send a signal to the auditory part of the brain, indicating a particular wavelength of sound has been heard. For a given tone, assume that the basilar fiber vibrates according to the equation:

$$z(t) = 15e^{-t/2}\sin(t/2),$$

where $z(t)$ measures the distance in microns from the rest position of the fiber and t is in milliseconds.

a. Determine all times when $z(t) = 0$ for $t \in [0, 4\pi]$.

b. Find the velocity of the basilar fiber by computing the derivative, $v(t) = z'(t)$.

c. Find the times $t \in [0, 4\pi]$ when the basilar fiber is at a maximum and a minimum. Give the values of z at these extrema. Sketch a graph of $z(t)$ for $t \in [0, 4\pi]$.

7. The muscles of the small intestine move chyme (food and enzymes) toward the colon in a process called *peristalsis* at a rate of 1-10 cm/sec. They periodically contract to create a traveling wave that causes the fluid in the small intestine to flow forward and allow absorption of nutrients into the blood. Consider a cross-section of the small intestine, assuming that it maintains a circular shape with a radius of $R(t)$ under the smooth muscle contraction. Suppose that the radius of one segment of the small intestine satisfies the function

$$R(t) = A + B\cos(\omega t),$$

where you must find the constants A, B, and ω.

a. While digesting food, assume that the segment of small intestine periodically contracts 10 times per minute. Assume that the maximum distention of this segment occurs at $t = 0$ and is 4 cm ($R(0) = 4$). When the muscle is maximally contracted, the opening of the segment in the intestine reduces to only a radius of 1 cm (the minimum of $R(t)$). With these data, find the constants A, B, and ω.

b. Sketch the graph of $R(t)$ for $t \in [0, 0.2]$. List all maxima and minima in the interval.

c. Differentiate $R(t)$ and find the maximum rate of decrease in the radius $R(t)$ (in cm/min) and the first time after $t = 0$ when this occurs.

8. In lab we considered a model for the length of day in San Diego. The effect is dramatically more pronounced when you go to Alaska. In Anchorage, Alaska, the longest day is June 20 at 19 hr 22 min or 1162 min. The shortest day is December 21 at 5 hr 27 min or 327 min. Consider a model for the length of the day in minutes, $L(t)$, as a function of the date, t, using the sine function as follows

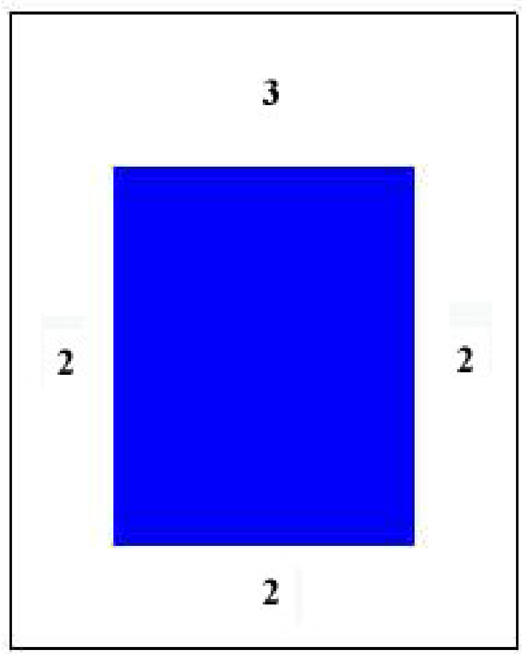
$$L(t) = \alpha + \beta\sin(\omega(t - \phi)),$$

where the constants α, β, ω, and ϕ are to be determined below (and assuming that January 1 is $t = 0$).

a. Assume that June 20 is given by day 170 with length of 1162 min. With the information that the shortest day is 327 min and a year is 365 days, find the constants α, β, ω, and ϕ. Write the function $L(t)$ and find the length of Ground Hog day (February 2 or Day 32) in Anchorage.

b. Differentiate $L(t)$ to find the rate of change in the length of day and write this formula. Find the date when this rate of change is increasing the most, and what is the rate of change per day at that date?

9. A brochure is to have an area of 125 in^2, with a 3 in margin at the top and 2 in margins on the sides and bottom. Find the dimensions of the brochure that allow the maximum printing area. (See figure below.)



10. Suppose that a study finds that the best fit for the number of drops required to break open a walnut satisfies the equation

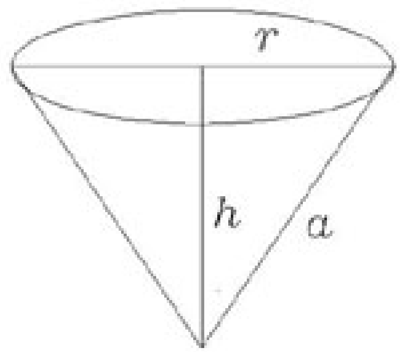
$$N(h) = 1 + \frac{10}{h-1},$$

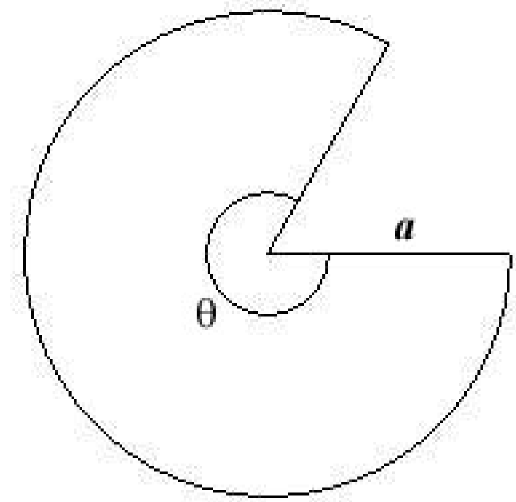
where h is the height of the drop in meters. The energy used to break open a walnut is given by the function

$$E(h) = hN(h).$$

Find the height that a crow should fly to minimize the energy needed to break open a walnut.

11. A water cup in the shape of a right circular cone is to be constructed by removing a sector from a circular sheet of paper of radius a and then joining the two straight edges of the remaining paper. Find the dimensions of the cup with the largest volume that can be constructed in this way. (Hint: From the diagrams below, the volume of the cone is $V = \pi r^2 h/3$, where the variables r and h satisfy $r^2 + h^2 = a^2$. Furthermore, the circumference of the base of the cone is equal to the length of the sector of the circle, so $2\pi r = a\theta$. Use this information to create a function for the volume of the cone that depends only on θ, then use standard optimization methods to find the maximum volume.)





12. When the nutrient is limited, a bacteria satisfies the logistic growth model:

$$B_{n+1} = B_n + 0.03B_n \left(1 - \frac{B_n}{500,000}\right), \qquad B_0 = 10,000,$$

where n is in minutes.

a. Find B_1 and B_2.

b. Determine the equilibria for this model. Find the stability of these equilibria and justify your stability argument by evaluating the derivative of the updating function.