1. a. The solution to the initial value problem,

$$\frac{dy}{dt} = 0.3y, \qquad y(0) = 20,$$

is

$$y(t) = 20e^{0.3t}.$$

It requires five steps to use Euler's method to approximate the solution using a stepsize of $h = 0.2$ for $t \in [0, 1]$. The Euler formula for this problem is

$$y_{n+1} = y_n + 0.2(0.3y_n) = y_n + 0.06y_n.$$

Below is a table showing the iterations for Euler's solution

t_n	y_n
$t_0 = 0$	$y_0 = 20$
$t_1 = 0.2$	$y_1 = y_0 + 0.06y_0 = 20 + 0.06(20) = 21.2$
$t_2 = 0.4$	$y_2 = y_1 + 0.06y_1 = 21.2 + 0.06(21.2) = 22.472$
$t_3 = 0.6$	$y_3 = y_2 + 0.06y_2 = 22.472 + 0.06(22.472) = 23.8203$
$t_4 = 0.8$	$y_4 = y_3 + 0.06y_3 = 23.8203 + 0.06(23.8203) = 25.2495$
$t_5 = 1.0$	$y_5 = y_4 + 0.06y_4 = 25.2495 + 0.06(25.2495) = 26.7645$

The exact solution for $t = 1$ is $y(1) = 26.9972$, so the percent error is $100\frac{26.7645-26.9972}{26.9972} = -0.86\%$.

3. a. To verify that $T(t) = 25 - t + 25e^{-0.2t}$ is a solution to the initial value problem

$$T' = -k(T - (20 - t)), \qquad T(0) = 50,$$

where $k = 0.2$ hr^{-1}, we must verify the initial condition and that the differential equation is satisfied. First, $T(0) = 25 - 0 + 25e^0 = 50$, so the initial condition is satisfied.

Next we differentiate the proposed solution, giving

$$T' = -1 + 25(-0.2)e^{-0.2t} = -1 - 5e^{-0.2t}.$$

For the right hand side of the equation, we substitute the solution giving

$$\begin{aligned} -k(T - (20 - t)) &= -0.2\left(25 - t + 25e^{-0.2t} - (20 - t)\right) \\ &= -0.2\left(5 + 25e^{-0.2t}\right) = -1 - 5e^{-0.2t}. \end{aligned}$$

Tbus, the equation is satisfied. The solution at $t = 2$ is $T(2) = 25 - 2 + 25e^{-0.4} = 39.76°$C.

b. For the Euler's solution, we need four steps to approximate the solution using a stepsize of $h = 0.5$ for $t \in [0, 2]$. The Euler formula for this problem is

$$\begin{aligned} T_{n+1} &= T_n + 0.5(-k(T_n - (20 - t_n))), \\ T_{n+1} &= T_n - 0.1(T_n + t_n - 20). \end{aligned}$$

Below is a table showing the iterations for this Euler's solution

t_n	T_n
$t_0 = 0$	$T_0 = 50$
$t_1 = 0.5$	$T_1 = T_0 - 0.1(T_0 + t_0 - 20) = 50 - 0.1(50 + 0 - 20) = 47$
$t_2 = 1.0$	$T_2 = T_1 - 0.1(T_1 + t_1 - 20) = 47 - 0.1(47 + 0.5 - 20) = 44.25$
$t_3 = 1.5$	$T_3 = T_2 - 0.1(T_2 + t_2 - 20) = 44.25 - 0.1(44.25 + 1 - 20) = 41.725$
$t_4 = 2.0$	$T_4 = T_3 - 0.1(T_3 + t_3 - 20) = 41.725 - 0.1(41.725 + 1.5 - 20) = 39.4025$

Since the solution at $t = 2$ is $T(2) = 39.76$, the error between Euler's method and the actual solution is $100\frac{39.4025-39.76}{39.76} = -0.90\%$.

5. a. The Euler's solution for this problem requires three steps to approximate the solution using a stepsize of $h = 1$ for $t \in [0, 3]$. The Euler formula for this problem is

$$\begin{aligned} R_{n+1} &= R_n + 1.0(-0.05R_n + 0.15e^{-0.02t_n}), \\ R_{n+1} &= R_n - 0.05R_n + 0.15e^{-0.02t_n}. \end{aligned}$$

Below is a table showing the iterations for this Euler's solution

t_n	R_n
$t_0 = 0$	$R_0 = 10$
$t_1 = 1$	$R_1 = R_0 - 0.05R_0 + 0.15e^{-0.02t_0} = 10 - 0.05(10) + 0.15e^{-0.02(0)} = 9.65$
$t_2 = 2$	$R_2 = R_1 - 0.05R_1 + 0.15e^{-0.02t_1} = 9.65 - 0.05(9.65) + 0.15e^{-0.02(1)} = 9.3145$
$t_3 = 3$	$R_3 = R_2 - 0.05R_2 + 0.15e^{-0.02t_2} = 9.3145 - 0.05(9.3145) + 0.15e^{-0.02(2)} = 8.9929$

Thus, at $t = 3$, the approximate solution is $R(3) = 8.99$.

b. To verify that $R(t) = 5e^{-0.05t} + 5e^{-0.02t}$ is a solution to the initial value problem

$$R' = -0.05R + 0.15e^{-0.02t}, \qquad R(0) = 10,$$

we must verify the initial condition and that the differential equation is satisfied. First, $R(0) = 5e^{-0.05(0)} + 5e^{-0.02(0)} = 10$, so the initial condition is satisfied.

Next we differentiate the proposed solution, giving

$$R' = -0.05\left(5e^{-0.05t}\right) - 0.02\left(5e^{-0.02t}\right) = -0.25e^{-0.05t} + -0.1e^{-0.02t}.$$

For the right hand side of the equation, we substitute the solution giving

$$\begin{aligned} -0.05R + 0.15e^{-0.02t} &= -0.05\left(5e^{-0.05t} + 5e^{-0.02t}\right) + 0.15e^{-0.02t} \\ &= -0.25e^{-0.05t} + (-0.25 + 0.15)e^{-0.02t} = -0.25e^{-0.05t} - 0.1e^{-0.02t}. \end{aligned}$$

Tbus, the equation is satisfied. The solution at $t = 3$ is $R(3) = 5e^{-0.05*3} + 5e^{-0.02*3} = 9.012363$. The percent error is therefore $100\frac{8.9929-9.0124}{9.0124} = -0.22\%$.