

1. a. The solution to the initial value problem is  $y(t) = 20e^{0.3t}$ . Below is a table for the Euler's solution.

|             |                 |
|-------------|-----------------|
| $t_0 = 0$   | $y_0 = 20$      |
| $t_1 = 0.2$ | $y_1 = 21.2$    |
| $t_2 = 0.4$ | $y_2 = 22.472$  |
| $t_3 = 0.6$ | $y_3 = 23.8203$ |
| $t_4 = 0.8$ | $y_4 = 25.2495$ |
| $t_5 = 1$   | $y_5 = 26.7645$ |

The solution at  $t = 1$  is  $y(1) = 26.9972$ , so the percent error between Euler's method and the actual solution is  $-0.86\%$ .

b. The solution to the initial value problem is  $y(t) = \frac{100}{3} - \frac{70}{3}e^{-0.3t}$ . Below is a table for the Euler's solution.

|             |                 |
|-------------|-----------------|
| $t_0 = 0$   | $y_0 = 10$      |
| $t_1 = 0.2$ | $y_1 = 11.4$    |
| $t_2 = 0.4$ | $y_2 = 12.716$  |
| $t_3 = 0.6$ | $y_3 = 13.9530$ |
| $t_4 = 0.8$ | $y_4 = 15.1159$ |
| $t_5 = 1$   | $y_5 = 16.2089$ |

The solution at  $t = 1$  is  $y(1) = 16.0476$ , so the percent error between Euler's method and the actual solution is  $1.0\%$ .

2. a. The solution to the initial value problem is  $P(t) = 20 + 80e^{0.1t}$ . The solution at  $t = 1$  is  $P(1) = 108.41$ .

b. Below is a table for the Euler's solution.

|             |                 |
|-------------|-----------------|
| $t_0 = 0$   | $P_0 = 100$     |
| $t_1 = 0.2$ | $P_1 = 101.6$   |
| $t_2 = 0.4$ | $P_2 = 103.232$ |
| $t_3 = 0.6$ | $P_3 = 104.897$ |
| $t_4 = 0.8$ | $P_4 = 106.595$ |
| $t_5 = 1$   | $P_5 = 108.326$ |

The percent error between Euler's method and the actual solution is  $-0.077\%$ .

3. a. If  $T(t) = 25 - t + 25e^{-0.2t}$ , then  $T'(t) = -1 - 5e^{-0.2t}$ . However,  $-k(T(t) - (20 - t)) = -0.2((25 - t + 25e^{-0.2t}) - (20 - t)) = -0.2(5 + 25e^{-0.2t}) = -1 - 5e^{-0.2t}$ . Since  $T(0) = 25 + 25e^{-0.2(0)} = 50$ , we have verified that we have the correct solution.  $T(2) = 39.76^\circ\text{C}$ .

b. Below is a table for the Euler's solution.

|             |                 |
|-------------|-----------------|
| $t_0 = 0$   | $T_0 = 50$      |
| $t_1 = 0.5$ | $T_1 = 47$      |
| $t_2 = 1$   | $T_2 = 44.25$   |
| $t_3 = 1.5$ | $T_3 = 41.725$  |
| $t_4 = 2$   | $T_4 = 39.4025$ |

The percent error between Euler's method and the actual solution is  $-0.90\%$ .

4. a. If  $T(t) = 20 - t + 20e^{-0.2t}$ , then  $T'(t) = -1 - 4e^{-0.2t}$ . However,  $-k(T(t) - (15 - t)) = -0.2((20 - t + 20e^{-0.2t}) - (15 - t)) = -0.2(5 + 20e^{-0.2t}) = -1 - 4e^{-0.2t}$ . Since  $T(0) = 20 + 20e^{-0.2(0)} = 40$ , we have verified that we have the correct solution.  $T(2) = 31.41^\circ\text{C}$ .

b. Below is a table for the Euler's solution.

|             |                |
|-------------|----------------|
| $t_0 = 0$   | $T_0 = 40$     |
| $t_1 = 0.5$ | $T_1 = 37.5$   |
| $t_2 = 1$   | $T_2 = 35.2$   |
| $t_3 = 1.5$ | $T_3 = 33.08$  |
| $t_4 = 2$   | $T_4 = 31.122$ |

The percent error between Euler's method and the actual solution is  $-0.91\%$ .

5. a. Below is a table for the Euler's solution.

|           |                |
|-----------|----------------|
| $t_0 = 0$ | $R_0 = 10$     |
| $t_1 = 1$ | $R_1 = 9.65$   |
| $t_2 = 2$ | $R_2 = 9.3145$ |
| $t_3 = 3$ | $R_3 = 8.9929$ |

b. If  $R(t) = 5e^{-0.05t} + 5e^{-0.02t}$ , then  $R'(t) = -0.25e^{-0.05t} - 0.1e^{-0.02t}$ . However,  $-0.05R(t) + 0.15e^{-0.02t} = -0.05(5e^{-0.05t} + 5e^{-0.02t}) + 0.15e^{-0.02t} = -0.25e^{-0.05t} - 0.1e^{-0.02t}$ . Since  $R(0) = 5e^{-0.05(0)} + 5e^{-0.02(0)} = 10$ , we have verified that we have the correct solution.  $R(3) = 9.0124$ , so the percent error between Euler's method and the actual solution is  $-0.22\%$ .