1. a.

$$\begin{aligned}\int_{-1}^{3}(2-x+x^2)dx &= \left(2x-\frac{x^2}{2}+\frac{x^3}{3}\right)\Big|_{-1}^{3}\\ &= \left(2(3)-\frac{3^2}{2}+\frac{3^3}{3}\right)-\left(2(-1)-\frac{(-1)^2}{2}+\frac{(-1)^3}{3}\right)=\frac{40}{3}.\end{aligned}$$

c.

$$\begin{aligned}\int_0^2 \left(x^2+1\right)^2 dx &= \int_0^2 \left(x^4+2x^2+1\right) dx\\ &= \left(\frac{x^5}{5}+\frac{2x^3}{3}+x\right)\Big|_0^2\\ &= \left(\frac{2^5}{5}+\frac{2(2)^3}{3}+2\right)-0=\frac{206}{15}.\end{aligned}$$

f.

$$\begin{aligned}\int_1^5 \frac{x^2+1}{x}dx &= \int_1^5 \left(x+\frac{1}{x}\right) dx\\ &= \left(\frac{x^2}{2}+\ln(x)\right)\Big|_1^5\\ &= \left(\frac{5^2}{2}+\ln(5)\right)-\left(\frac{1^2}{2}+\ln(1)\right)=12+\ln(5).\end{aligned}$$

g.

$$\begin{aligned}\int_0^{\pi} (\cos(2t)+4t)\, dt &= \left(\frac{\sin(2t)}{2}+\frac{4t^2}{2}\right)\Big|_0^{\pi}\\ &= \left(\frac{\sin(2\pi)}{2}+2\pi^2\right)-\left(\frac{\sin(0)}{2}+0\right)=2\pi^2.\end{aligned}$$

i. This problem requires a substitution. We choose the quantity under the square root, so we let $u = 25 - x^2$, which implies that $du = -2xdx$. The limits of integration must also be changed. Since $x = 3$, it follows that $u = 25 - 3^2 = 16$, and similarly since $x = 4$, we have $u = 25 - 4^2 = 9$. With these substitutions, we have

$$\begin{aligned}\int_3^4 \frac{2x}{\sqrt{25-x^2}}dx &= \int_{16}^{9} \frac{-du}{\sqrt{u}} = -\int_{16}^{9} u^{-1/2}du\\ &= \left(-\frac{u^{1/2}}{1/2}\right)\Big|_{16}^{9} = -2u^{1/2}\Big|_{16}^{9}\\ &= -2\left(9^{1/2}\right)+2\left(16^{1/2}\right)=2.\end{aligned}$$

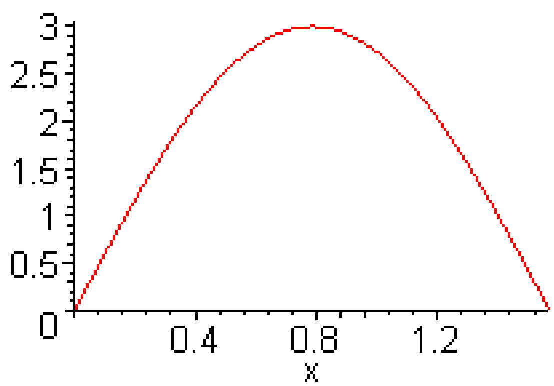
1. This problem requires a substitution. We choose the squared quantity, so we let $u = \sin(x)+1$, which implies that $du = \cos(x)dx$. The lower limit of integration is $x = 0$, it follows that $u = \sin(0)+1 = 1$. At the upper limit, we have $x = \pi/2$, so $u = \sin(\pi/2)+1 = 2$. With these substitutions, we have

$$\begin{aligned}\int_0^{\pi/2} 3\left(\sin(x)+1\right)^2\cos(x)dx &= \int_1^2 3u^2du \\ &= \left(3\frac{u^3}{3}\right)\Big|_1^2 \\ &= 2^3 - 1^3 = 7.\end{aligned}$$

3. The area is given by the definite integral

$$\begin{aligned}A &= \int_0^{\pi/2} 3\sin(2x)dx \\ &= \left(-3\frac{\cos(2x)}{2}\right)\Big|_0^{\pi/2} \\ &= \left(-3\frac{\cos(\pi)}{2}\right) + \left(3\frac{\cos(0)}{2}\right) = 3.\end{aligned}$$

The graph is shown below.

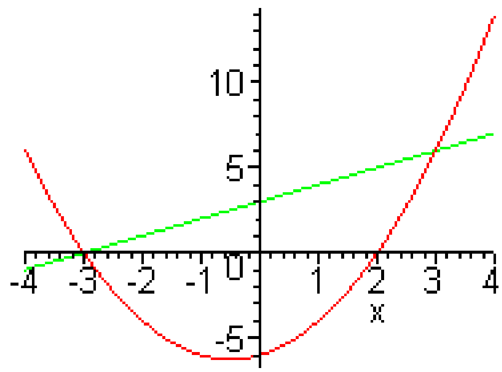


4. a. For the line $y = x+3$, the x and y-intercepts are $(-3, 0)$ and $(0, 3)$, respectively. (These are easily seen by first solving for $y = 0$ for the x-intercept, then setting $x = 0$ for the y-intercept. For the quadratic, the x-intercepts are found by solving $x^2 + x - 6 = (x+3)(x-2) = 0$, so $x = -3$ and 2, which gives the x-intercepts as $(-3, 0)$ and $(2, 0)$. Once again the y-intercept is found by letting $x = 0$, so $y = -6$. The vertex occurs halfway between the x-intercepts, which is at $x = -\frac{1}{2}$. By putting this into the quadratic function, we get the y value, so the vertex is at $(-\frac{1}{2}, -6\frac{1}{4})$. The graph is shown below.

b. The x values of the points of intersection are found by setting the two equations equal to each other, so $x^2 + x - 6 = x + 3$ or $x^2 - 9 = 0$, which gives $x = \pm 3$. It easily follows by substituting back into the equation for the line that the points of intersection are $(-3,0)$ and $(3,6)$.

c. Since the line is above the parabola, the area is found by choosing an integrand with the equation of the quadratic subtracted from the equation of the line. The limits of integration are determined by the x values of the points of intersection. Thus, area satisfies the definite integral

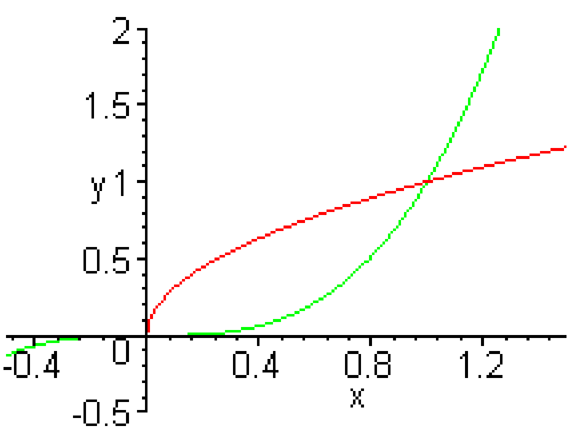
$$\begin{aligned} A &= \int_{-3}^{3} \left((x+3) - (x^2+x-6)\right) dx = \int_{-3}^{3} \left(9 - x^2\right) dx \\ &= \left(9x - \frac{x^3}{3}\right)\Big|_{-3}^{3} \\ &= \left(9(3) - \frac{3^3}{3}\right) - \left(9(-3) - \frac{(-3)^3}{3}\right) = 36. \end{aligned}$$



6. a. Since both of these functions are just powers of x, then their graphs must pass through the origin and the point $(1,1)$. Thus, the points of intersection are $(0,0)$ and $(1,1)$. The graph is shown below.

b. On the interval $x \in [0,1]$, the function $y = \sqrt{x}$ is greater than the function $y = x^3$. Thus, the area is given by the formula

$$\begin{aligned} A &= \int_{0}^{1} \left(\sqrt{x} - x^3\right) dx \\ &= \left(\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right)\Big|_{0}^{1} \\ &= \left(\frac{2}{3} - \frac{1}{4}\right) - 0 = \frac{5}{12}. \end{aligned}$$



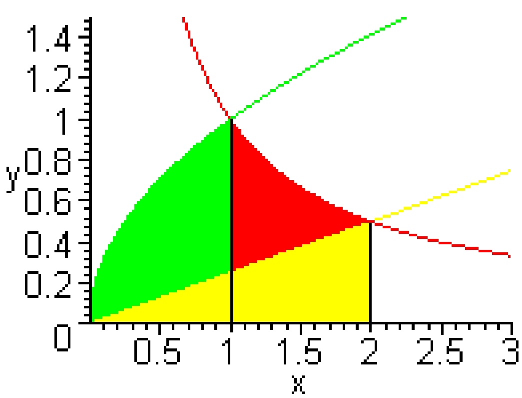
8. The graph is divided up as below showing the the distinct regions needed for computing the area. First, we need to find the points of intersection. The curves $y = \sqrt{x}$ and $y = x/4$ intersection at the origin very clearly. Then the curve $y = 1/x$ enters and intersects $y = \sqrt{x}$. The x-value of this intersection satisfies $\sqrt{x} = \frac{1}{x}$ or $x = 1$. Finally, the curve $y = 1/x$ decreases until it intersects the curve $y = x/4$. These curves intersect when $\frac{1}{x} = \frac{x}{4}$, so $x^2 = 4$ or $x = 2$. The points of intersection are $(0, 0)$, $(1, 1)$, and $(2, \frac{1}{2})$.

As seen in the graph, the area must be computed by two distinct integrals. The first region has the curve $y = \sqrt{x}$ above the curve $y = x/4$ (for $0 \le x \le 1$), while the second region has the curve $y = 1/x$ above the curve $y = x/4$ (for $1 \le x \le 2$). The area is computed by evaluating the integrals

$$\int_0^1 \left(\sqrt{x} - \frac{x}{4}\right) dx + \int_1^2 \left(\frac{1}{x} - \frac{x}{4}\right) dx$$

We begin by expanding the integrals, which gives

$$\begin{aligned}
\int_0^1 \left(\sqrt{x} - \frac{x}{4}\right) dx + \int_1^2 \left(\frac{1}{x} - \frac{x}{4}\right) dx &= \int_0^1 x^{1/2} dx - \int_0^1 \frac{x}{4} dx + \int_1^2 \frac{1}{x} dx - \int_1^2 \frac{x}{4} dx \\
&= \int_0^1 x^{1/2} dx + \int_1^2 \frac{1}{x} dx - \int_0^2 \frac{x}{4} dx \\
&= \left.\frac{x^{3/2}}{3/2}\right|_0^1 + \left.\ln(x)\right|_1^2 - \left.\frac{1}{4}\left(\frac{x^2}{2}\right)\right|_0^2 \\
&= \left(\frac{2}{3} - 0\right) + (\ln(2) - \ln(1)) - \left(\frac{2^2}{8} + 0\right) \\
&= \frac{2}{3} - \frac{1}{2} + \ln(2) = \frac{1}{6} + \ln(2).
\end{aligned}$$



9. a. The extrema are found where the derivative is zero, so

$$\frac{dP}{dt} = t^3 - 9t^2 + 18t = t(t-3)(t-6) = 0,$$

or $t = 0$, 3, and 6. Since $P(t) = \frac{1}{4}t^4 - 3t^3 + 9t^2 + 12$, it follows that $P(0) = 12$, $P(3) = \frac{3^4}{4} - 3(3)^3 + 9(3)^2 + 12 = \frac{129}{4}$, and $P(6) = \frac{6^4}{4} - 3(6)^3 + 9(6)^2 + 12 = 12$. Thus, there are two minima at $(0, 12)$ and $(6, 12)$ and one maximum at $(3, 32\frac{1}{4})$.

b. The average population is given by

$$\begin{aligned} P_{ave} &= \frac{1}{7}\int_0^7 P(t)dt \\ &= \frac{1}{7}\int_0^7 \left(\frac{1}{4}t^4 - 3t^3 + 9t^2 + 12\right) dt \\ &= \frac{1}{7}\left(\frac{t^5}{20} - \frac{3t^4}{4} + \frac{9t^3}{3} + 12t\right)\Big|_0^7 \\ &= \frac{1}{7}\left(\frac{7^5}{20} - \frac{3(7)^4}{4} + 3(7)^3 + 12(7)\right) \\ &= \frac{152.6}{7} = 21.8 \end{aligned}$$

c. We know that the sine and cosine functions achieve a maximum value of 1 and a minimum value of -1. The cosine function is 1 when its argument is any even multiple of π, and it is -1 when its argument is any odd multiple of π. For $Q(t)$, the minima will occur when the argument $\pi t/3 = 0$ or 2π (since this substracts the most from Q). These occur when $t = 0$ or 6. The maximum of Q occurs when $\pi t/3 = \pi$ or $t = 3$ (since this adds the most to Q). (Alternately, the extrema are found where the derivative is zero or $\frac{dQ}{dt} = \frac{10\pi}{3}\sin(\frac{\pi}{3}t)$. This implies that $\sin(\frac{\pi}{3}t) = 0$, which gives the same values of t as above.)

Since $Q(0) = 22 - 10 = 12$ and $Q(6) = 22 - 10 = 12$, the two minima are at $(0, 12)$ and $(6, 12)$. Because $Q(3) = 22 + 10 = 32$, the maximum occurs at $(3, 32)$. The short solutions have a graph of both $P(t)$ and $Q(t)$ with the data.

d. The average population is given by

$$\begin{aligned} Q_{ave} &= \frac{1}{7}\int_0^7 Q(t)dt = \frac{1}{7}\int_0^7 \left(22 - 10\cos(\frac{\pi}{3}t)\right) dt \\ &= \frac{1}{7}\left(22t - 10\frac{3}{\pi}\sin(\frac{\pi}{3}t)\right)|_0^7 \\ &= \frac{1}{7}\left(22(7) - \frac{30}{\pi}\sin(\frac{7\pi}{3})\right) \\ &= 22 - \frac{30}{7\pi}\left(\frac{\sqrt{3}}{2}\right) = 20.82 \end{aligned}$$

11. a. This is a standard radioactive decay problem, so the solution is $R(t) = 50\,e^{-0.1t}$. The half-life is when the level reaches 25 mCi, so solving $25 = 50\,e^{-0.1t}$, we see $e^{0.1t} = 2$ or $0.1\,t = \ln(2)$. It follows that the half-life is $t = 10\,\ln(2) \simeq 6.9$ days.

b. The total exposure is given by

$$\begin{aligned} \int_0^{10} 5\,e^{-0.1t}dt &= -50\,e^{-0.1t}\Big|_0^{10} \\ &= 50\left(1 - e^{-1}\right) \simeq 31.6 \text{ mCi} \end{aligned}$$

c. For exposure less than 10 mCi, we solve the equation

$$\begin{aligned} \int_0^{t} 5\,e^{-0.1s}ds &= 10 \\ -50\,e^{-0.1s}\Big|_0^{t} &= 10 \\ 50\left(1 - e^{-0.1t}\right) &= 10 \\ e^{-0.1t} &= 0.8 \\ -0.1\,t &= \ln(0.8) \\ t &= 10\,\ln\left(\frac{5}{4}\right) \simeq 2.23 \text{ days.} \end{aligned}$$

Thus, the time of exposure is $t \le 10\,\ln(5/4) \simeq 2.23$ days.

13. To find the expected value of $x \in [0, 2]$ for $\sigma = 1$, we evaluate

$$\frac{1}{\sqrt{2\pi}} \int_0^2 x\,e^{-x^2/2}dx.$$

This integral is readily evaluated by making the substitution $u = -x^2/2$ or $du = -x\,dx$. The endpoints change from $x = 0$ and 2 to $u = 0$ and -2. It follows that

$$\begin{aligned} \frac{1}{\sqrt{2\pi}} \int_0^2 x\,e^{-x^2/2}dx &= -\frac{1}{\sqrt{2\pi}} \int_0^{-2} e^u du \\ &= -\frac{1}{\sqrt{2\pi}} e^u\Big|_0^{-2} \\ &= \frac{1}{\sqrt{2\pi}}\left(1 - e^{-2}\right) \simeq 0.345 \end{aligned}$$

The expected value of x is $x_m = (1 - e^{-2})/\sqrt{2\pi} \simeq 0.345$.