1. b.

$$\begin{aligned}\int \left(2t - \frac{6}{t^2} + \cos(4t)\right) dt &= \int 2t dt - 6\int t^{-2} dt + \int \cos(4t) dt \\ &= \frac{2t^2}{2} - \frac{6t^{-1}}{-1} + \frac{\sin(4t)}{4} + C \\ &= t^2 + \frac{6}{t} + \frac{\sin(4t)}{4} + C\end{aligned}$$

c.

$$\begin{aligned}\int (2 - \sqrt{x} + 4x^{1/3}) dx &= 2\int 1 dx - \int x^{1/2} dx + 4\int x^{1/3} dx \\ &= 2x - \frac{x^{3/2}}{3/2} + \frac{4x^{4/3}}{4/3} + C \\ &= 2x - \frac{2}{3}x^{3/2} + 3x^{4/3} + C\end{aligned}$$

g.

$$\begin{aligned}\int \left(6t^5 - \frac{4}{t^3} + \frac{2}{e^{2t}}\right) dt &= 6\int t^5 dt - 4\int t^{-3} dt + 2\int e^{-2t} dt \\ &= \frac{6t^6}{6} - \frac{4t^{-2}}{-2} + \frac{2e^{-2t}}{-2} + C \\ &= t^6 + 2t^{-2} - e^{-2t} + C\end{aligned}$$

h.

$$\begin{aligned}\int \left((t^3+1)^2 - \cos(2t)\right) dt &= \int \left(t^6 + 2t^3 + 1 - \cos(2t)\right) dt \\ &= \int t^6 dt + \int 2t^3 dt + \int 1 dt - \int \cos(2t) dt \\ &= \frac{t^7}{7} + \frac{2t^4}{4} + t - \frac{\sin(2t)}{2} + C \\ &= \frac{t^7}{7} + \frac{t^4}{2} + t - \frac{1}{2}\sin(2t) + C\end{aligned}$$

2. a.Since $\frac{dy}{dt} = 2 - 0.1e^{-t}$, we integrate to obtain

$$y(t) = \int \left(2 - 0.1e^{-t}\right) dt = 2t - \frac{0.1e^{-t}}{-1} + C.$$

From the initial condition, $y(0) = 10 = 0 + 0.1 + C$, so $C = 10 - 0.1 = 9.9$. Thus, the solution is given by

$$y(t) = 2t + 0.1e^{-t} + 9.9.$$

c. Since $\frac{dy}{dt} + 2y = 2$ can be written $\frac{dy}{dt} = -2(y-1)$, our techniques from the linear section apply. Let $z(t) = y(t) - 1$. Since $y(0) = 5$, $z(0) = 5 - 1 = 4$. Thus, we solve the equation

$$\frac{dz}{dt} = -2z, \qquad z(0) = 4,$$

which has the solution, $z(t) = 4e^{-2t} = y(t) - 1$. So

$$y(t) = 4e^{-2t} + 1.$$

f. Since $\frac{dy}{dt} = \cos(2t) + 2t$, we integrate to obtain

$$y(t) = \int (\cos(2t) + 2t)\, dt = \frac{\sin(2t)}{2} + t^2 + C.$$

From the initial condition, $y(0) = 4 = \sin(0) + 0 + C$, so $C = 4$. Thus, the solution is given by

$$y(t) = \frac{\sin(2t)}{2} + t^2 + 4.$$

h. Since $\frac{dh}{dt} = \frac{2}{t}$, we integrate to obtain

$$h(t) = 2\int t^{-1} dt = 2\ln(t) + C.$$

From the initial condition, $h(1) = 3 = 2\ln(1) + C$, so $C = 3$. Thus, the solution is given by

$$h(t) = 2\ln(t) + 3.$$

4. a. The differential equation for the velocity is given by

$$\frac{dv}{dt} = -32 \qquad v(0) = 96.$$

Upon integration, this has the solution $v(t) = -32t + C_v$. Since $v(0) = 0 + C_v = 96$,

$$v(t) = 96 - 32t.$$

The differential equation for the height of the ball satisfies

$$dh/dt = v(t) = 96 - 32t \qquad h(0) = 256.$$

Upon integration, this has the solution $h(t) = 96t - \frac{32t^2}{2} + C_h$. From the initial height, $h(0) = 256 = 0 + 0 + C_h$, so $C_h = 256$ and the solution becomes

$$h(t) = 256 + 96t - 16t^2.$$

b. At the maximum height, $h'(t) = 0 = v(t)$ or $96 - 32t = 0$, which occurs at $t = 3$ seconds. The height satisfies $h(3) = 256 + 96(3) - 16(9) = 400$ ft. When it hits the ground, $h(t) = 0$. But

$$h(t) = -16(t^2 - 6t - 16) = -16(t - 8)(t + 2).$$

Thus, it hits the ground when $t = 8$ sec (the solution at $t = -2$ doesn't make sense). The velocity of impact is $v(8) = -32(8) + 96 = -160$ ft/sec.

5. The differential equation for the velocity is given by

$$\frac{dv}{dt} = -32 \qquad v(0) = v_0,$$

where v_0 is to be determined. Upon integration, this has the solution $v(t) = -32t + C_v$. Since $v(0) = 0 + C_v = v_0$,

$$v(t) = v_0 - 32t.$$

The differential equation for the height of the ball satisfies

$$dh/dt = v(t) = v_0 - 32t \qquad h(0) = 0.$$

Upon integration, this has the solution $h(t) = v_0 t - 16t^2 + C_h$. From the initial height, $h(0) = 0 = 0 + 0 + C_h$, so $C_h = 0$ and the solution becomes

$$h(t) = v_0 t - 16t^2.$$

At its maximum height, the velocity of the antelope is zero or $\frac{dh}{dt} = v(t) = 0$. Thus, $0 = v_0 - 32t$ or $t = v_0/32$. This is put into the equation for h, giving

$$h(v_0/32) = 6 = \frac{v_0^2}{32} - \frac{16v_0^2}{32^2} = \frac{v_0^2}{64}.$$

This is solved for v_0, so $v_0^2 = 384$ or $v_0 = \sqrt{384} \simeq 19.6$ ft/sec. To find the time in the air, we solve $h(t) = 0 = v_0 t - 16t^2 = t(v_0 - 16t)$. Thus, $t = v_0/16 \simeq 1.225$ sec.

7. a. This problem uses down for the positive direction. The differential equation for the velocity satisfies

$$\frac{dv}{dt} = 9.8 \qquad v(0) = 0.$$

Upon integration, this has the solution $v(t) = 9.8t + C_v$. Since $v(0) = 0 + C_v = 0$,

$$v(t) = 9.8t.$$

The differential equation for the height of the ball satisfies

$$ds/dt = v(t) = 9.8t \qquad s(0) = 0.$$

Upon integration, this has the solution $s(t) = \frac{9.8t^2}{2} + C_s$. From the initial height, $s(0) = 0 = 0 + 0 + C_s$, so $C_s = 0$ and the solution becomes

$$s(t) = 4.9t^2.$$

The fall lasts until $s(t_g) = 4.9t_g^2 = 39.2$ m, so $t_g = \sqrt{\frac{39.2}{4.9}} = 2\sqrt{2} \simeq 2.828$ sec. The velocity of impact is given by $v(t_g) = 9.8t_g \simeq 27.72$ m/sec.

b. The differential equation with air resistance is given by

$$\frac{dv}{dt} = g - kv = -0.35(v - 28) \quad \text{with} \quad v(0) = 0.$$

This is solved by making the substitution $z(t) = v(t) - 28$ with $z(0) = v(0) - 28 = -28$. The solution to this problem is $z(t) = -28e^{-0.35t} = v(t) - 28$, so

$$v(t) = 28(1 - e^{-0.35t}).$$

The velocity at $t = 3$ sec is $v(3) = 28(1 - e^{-1.05}) \simeq 18.2$ m/sec.

c. The position $s(t)$ (with $s(0) = 0$) satisfies the differential equation

$$\frac{ds}{dt} = v(t) = 28 - 28e^{-0.35t}.$$

This is integrated, so

$$\begin{aligned} s(t) &= \int \left(28 - 28e^{-0.35t}\right) dt = 28t - \frac{28e^{-0.35t}}{-0.35} + C_s \\ &= 28t + 80e^{-0.35t} - 80, \end{aligned}$$

since $s(0) = 0 = 0 + 80 + C_s$. At $t = 3$, $s(3) = 28(3) + 80e^{-1.05} - 80 \simeq 32.0$ m.

d. The Newton's formula is given by $t_{n+1} = t_n - \frac{f(t_n)}{f'(t_n)}$, where

$$f(t) = s(t) - 39.2 = 28t + 80e^{-0.35t} - 119.2,$$

with $f'(t) = 28 - 28e^{-0.35t}$. Thus, Newton's formula becomes

$$t_{n+1} = t_n - \frac{28t_n + 80e^{-0.35t_n} - 119.2}{28 - 28e^{-0.35t_n}}.$$

Performing two Newton's iterates we see that

$$t_1 = 3 - \frac{28(3) + 80e^{-1.05} - 119.2}{28 - 28e^{-1.05}} = 3.3958 \text{ sec}$$

$$t_2 = 3.3958 - \frac{28(3.3958) + 80e^{-1.189} - 119.2}{28 - 28e^{-1.189}} = 3.3827 \text{ sec.}$$

The velocity with which the cat hits the ground is $v(3.3827) = 28 - 28e^{-0.35(3.3827)} = 19.43$ m/sec.

8. a. The weight of a walleye is given by the differential equation:

$$\frac{dw}{dt} = 0.03(10 - w) = -0.03(w - 10),$$

so we make the substitutions $z(t) = w(t) - 10$, $z'(t) = w'(t)$, and $z(0) = w(0) - 10 = -10$. Thus, we solve the initial value problem

$$\frac{dz}{dt} = -0.03z, \qquad z(0) = -10,$$

which has the solution $z(t) = -10e^{-0.03t} = w(t) - 10$. Thus,

$$w(t) = 10 - 10e^{-0.03t}.$$

The time for a 2 kg walleye satisfies the equation $2 = 10 - 10e^{-0.03t}$, so $8 = 10e^{-0.03t}$ or $e^{0.03t} = \frac{5}{4}$ By taking logarithms, we have $0.03t = \ln\left(\frac{5}{4}\right)$ or $t = \frac{100}{3}\ln\left(\frac{5}{4}\right) \simeq 7.4$ yr. If you let t get very large, then $w(t) \to 10$ kg, since the exponential decays to zero.

b. With the result from Part a., the differential equation for the amount of PCBs in the walleye is given by

$$\frac{dP}{dt} = 0.3w(t) = 3 - 3e^{-0.03t}.$$

Thus, by integrating we find $P(t)$, so

$$P(t) = \int \left(3 - 3e^{-0.03t}\right) dt = 3t - 3\frac{e^{-0.03t}}{-0.03} + C = 3t + 100e^{-0.03t} + C.$$

From the initial condition, $P(0) = 0 = 3(0) + 100e^0 + C$, so $C = -100$. Thus, the solution of the differential equation becomes

$$P(t) = 3t + 100e^{-0.03t} - 100.$$

We use this formula to enter the values of $t = 2$, 5, and 10 to get $P(2) = 0.176$ mg, $P(5) = 1.071$ mg, and $P(10) = 4.082$ mg.

c. This part of the problem is easily solved by first substituting into the weight equation from Part a at the times 2, 5, and 10 years to give $w(2) = 0.582$ kg, $w(5) = 1.393$ kg, and $w(10) = 2.592$ kg. Next we use the formula $c(t) = P(t)/w(t)$ with the values for $P(t)$ in Part b and $w(t)$ just calculated to obtain $c(2) = 0.303$ μg/g, $c(5) = 0.769$ μg/g, and $c(10) = 1.575$ μg/g.