Computer Lab Problem: Cadmium Exposure and Smoking

- Discuss dangers and sources of Cadmium
- Basic **differential equation** for Cadmium accumulation in the kidney
- Find parameters to fit data
- Use **integrals** to determine exposure
- Modify to include increased risk of smoking
- Show numerical integration techniques

Computer Lab Problem: Cadmium Exposure

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Cadium is a toxic heavy metal used in nickel-cadium batteries and cadium telluride solar panels. However, because of its toxicity its use has significantly decreased in other applications. Human exposure to cadium (Cd) comes from two primary sources. It can be ingested, often with leafy vegetables, raw potatoes, and certain meats, where about 0.5-1.0 μg/day are retained. It is much more readily absorbed through the lungs from cigarette smoke, often doubling the intake in the body. The metal concentrates in the kidney tissue. High exposure can cause itai-itai disease and renal failure (cadium poisoning). Lower exposure has been linked to the increased risk of cancer (cadium and smoking).

a. Cadium is poorly removed from the body and accumulates in the kidney. A differential equation describing the amount of Cd, $C(t)$, in the kidney of a nonsmoker (in mg) is given by:

$$\frac{dC}{dt} = A - kC, \qquad C(0) = 0,$$

where A represents the amount of Cd entering by ingestion of food, k represents the removal rate, and t is in years. Find the solution of this differential equation in terms of A and k.

$C(t) =$ (A/k)*(1-exp(-k*t))

b. Below are data for the total Cd in the kidney (in mg) for an average nonsmoker at different ages [1].

Age (yr)	6	16	25	34	45	57
$C(t)$	0.28	0.81	1.35	1.54	1.66	1.72

Find the least squares best fit of the data to the solution of the differential equation above. Give the values of the constants A and k and write the model with these constants. Include the value of the least sum of squares error fitting the data.

$A =$ 0.07547592
$k =$ 0.037000869
$C(t) =$ 2.039841831 *(1-exp(-0.0370009*t)) mg
$SSE =$ 0.051991875

c. The risk of cancer from cadium is computed by the exposure to this element. The exposure, $E(t)$, is found by the amount of Cd in the tissue times the amount of time that it remains in the tissue. This is readily computed by the integral, which is given by:

$$E(t) = \int_0^t C(s)ds.$$

Lab Problem: Cadmium Exposure with Smoking

d. In your Lab report, create a graph with the data and the Cadium model for $t \in [0, 70]$. Briefly describe how well the model simulates the data. Create a second graph of the model of exposure to Cd for $t \in [0, 70]$for an average nonsmoker. Briefly describe what this graph is saying about the risk of cancer from Cd for a nonsmoker as someone ages.

e. As noted above, lungs absorb cadium much more readily than the gut, so the Cd in cigarettes can easily double the intake of Cd. Because of the carcinogenic properties of Cd, this further increases the cancer risk from smoking. Assume that a smoker begins at age 18. As a simplifying assumption, we will assume that the smoker smokes the same amount of cigarettes annually, and that this increases the Cd intake by a factor of 1.85. For the first 18 years, the amount of Cd entering the body of the smoker is the same as the nonsmoker, following the differential equation in Part a above. For the remainder of the time in this problem, the differential equation describing the amount of Cd, $C_1(t)$, in the kidney of the smoker (in mg) satisfies:

$$\frac{dC_1}{dt} = 1.85A - kC_1, \qquad C_1(18) = C(18),$$

where A and k are the values calculated above. You compute $C(18)$ using your solution from Part a. Find the solution of this initial value problem for $t \geq 18$.

$C_1(t) =$ 3.773708207-5.414755992*exp(-0.0370009*t)

f. Again, the exposure, $E_1(t)$, is found by the amount of Cd in the tissue times the amount of time that it remains in the tissue. The first 18 years are found with the same formula as given in Part c, so $E_1(t) = E(t)$. However, the increased Cd in tobacco results in a new formula for $E_1(t)$ for $t \geq 18$. This is computed by the integral, which is given by:

$$E_1(t) = \int_0^{18} C(s)ds + \int_{18}^{t} C_1(s)ds.$$

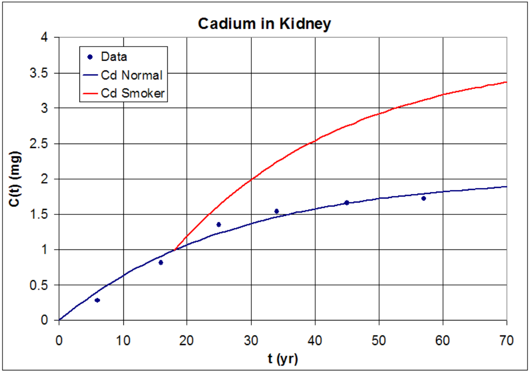
$E_1(t) =$ -133.1992197+3.773708208*t+146.3411969*exp(-.0370009*t) for $t \geq 18$.

Use this formula and the models, $C(t)$ and $C_1(t)$, to determine the exposure of this smoker at ages 30, 50, and 70. Find the exact value of the integral, then use both the Midpoint and Trapezoid Rules with a stepsize of $h = 2$ to approximate all of the integrals.

At age 30, the exact value is

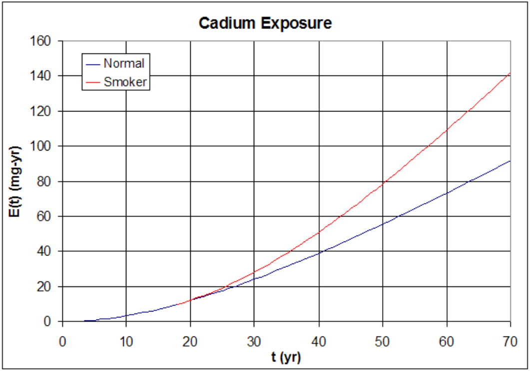
$E_1(30) =$ ______ mg-yr

Computer Lab Problem: Cadmium Accumulation



- Students fit data from non-smoker – food intake
- See increased accumulation through lungs by smoking

Computer Lab Problem: Cadmium Exposure/Risk



- Graphs show spreading risk factor with age
- Observe smoker has same risk at about 47 as a 60 year old non-smoker

Learning Objectives from Cadmium Exposure and Smoking

- Linear Differential Equation
  - Seen in Models for Newton's Law of Cooling and Lake Pollution
  - Fit Data – Exponentials and Horizontal Asymptotes
  - Extending Solutions – Smoking starting at Age X
- Integration
  - Similar to Lead and Mercury Exposure Problems
  - Show Numerical Integration "close" to actual integral
- Modeling helps explain increased risk of smoking