1 (a) $\hat{y} = 20.25 + 0.19x_1 + 0.54x_2$

(b) $\hat{y}_i = 20.25 + 0.19(65) + 0.54(74) = 72.56$

$e_i = y_i - \hat{y}_i = 71 - 72.56 = -1.56$

(c) $\hat{\beta}_2 = 0.54$

(d) $\hat{\beta}_1 \pm t(n-k-1)_{\alpha/2} \, SE_{\hat{\beta}_1} = 0.19 \pm 1.99(0.088) = 0.19 \pm .175 = (.015, .365)$ $\quad df = 73$

We are 95% confident that, for each inch increase in mother's height, student height increases, on average, by .015 to .365 inches, comparing students w/ fathers having the same height.
Because the C.I. does not include 0, mother's height is useful for predicting student height.

(e) H_0: $\beta_1 = \beta_2 = 0$ vs. H_a: $\beta_1 \neq 0$ and/or $\beta_2 \neq 0$

$F = 33.86$ $\quad p \approx 0 < \alpha = 0.05 \Rightarrow$ reject H_0

Mother's height and father's height together have statistically significant predictive value.

(f)
$$R^2 = \frac{SSR}{SST} = \frac{454.71}{944.85} = 0.481$$

About 48.1% of the total variation in student height is explained by mother's and father's heights.

(g) 95% C.I. for $E(y_{n+1})$: (69.976, 71.365)

We are 95% confident that the mean height of many students with mother's height = 63 inches and father's height = 71 inches will be between 69.976 and 71.365 inches.

95% P.I. for y_{n+1}: (65.460, 75.881)

We are 95% confident that the height of a student with mother's height = 63 inches & father's height = 71 inches will be between 65.46 & 75.881 inches.

2.(a) $S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 509.12 - \frac{100^2}{20} = 9.12$

$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n} = 257.66 - \frac{100 \times 50}{20} = 7.66$

$\bar{x} = \Sigma x / n = 100/20 = 5$

$\bar{y} = \Sigma y / n = 50/20 = 2.5$

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{7.66}{9.12} = 0.84$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 2.5 - 0.84(5) = -1.7$

The L.S. prediction equation is $\hat{y} = -1.7 + 0.84x$

Based on this equation, a one-point increase in the entrance test score is associated with a 0.84 point increase in the freshman GPA.

(b) $H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 134.84 - \frac{50^2}{20} = 9.84$

$S_\varepsilon = \sqrt{\frac{S_{yy} - (S_{xy})^2 / S_{xx}}{n-2}} = \sqrt{\frac{9.84 - \frac{(7.66)^2}{9.12}}{20-2}} = 0.435$

$SE_{\hat{\beta}_1} = S_\varepsilon / \sqrt{S_{xx}} = 0.435 / \sqrt{9.12} = 0.144$

$t = \frac{\hat{\beta}_1 - 0}{SE_{\hat{\beta}_1}} = \frac{0.84 - 0}{0.144} = 5.83$

$df = n - 2 = 18.$

RR reject H_0 if $|t| > t_{\alpha/2}(18) = 2.101$

Since $5.83 > 2.101 \Rightarrow$ reject H_0.

2(b) continued

<u>OR</u> p-value $= 2P(t(18) > 5.83)$

$< 2 \times .0005 = .001 < \alpha = 0.05 \Rightarrow$ reject H_0.

There is a linear relationship between entrance test score and freshman GPA.

(c) $x_{n+1} = 4.7$

$$\hat{y}_{n+1} = -1.7 + 0.84(4.7) = 2.248$$

95% prediction interval for y_{n+1}

$$\hat{y}_{n+1} \pm t_{\alpha/2}(n-2)\, s_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_{n+1} - \bar{x})^2}{S_{xx}}}$$

$$= 2.248 \pm 2.101(0.435)\sqrt{1 + \frac{1}{20} + \frac{(4.7-5)^2}{9.12}}$$

$$= 2.248 \pm 0.941 \approx (1.31, 3.19)$$

We are 95% confident that Mary Jones' freshman GPA will be between 1.31 and 3.19.

(d) The C.I. for $E y_{n+1}$ would be narrower than the P.I. for y_{n+1} because the C.I. is for ~~one~~ average freshman GPA for all students whose entrance score is 4.7 and the P.I. is for one particular student whose entrance score is 4.7. There is more variability in a single y_{n+1} than in $E y_{n+1}$