15.10

a. $y_{ij} = \mu + \alpha_k + \beta_i + \gamma_j + \epsilon_{ij}$; $i, j, k = 1, 2, 3, 4$;
where y_{ij} is the mileage of a Driver i in Car Model j.
α_k is the effect of the kth Gasoline Blend on mileage
β_i is the effect of the ith Driver on mileage
γ_j is the effect of the jth Car Model on mileage

b. The Row, Column, and Treatment Means are given here:

Level	1	2	3	4
Driver Mean $\bar{Y}_{i.}$	23.05	21.08	22.40	22.45
Model Mean $\bar{Y}_{.j}$	14.08	31.55	17.50	25.85
Blend Mean $\bar{Y}_k$	22.50	24.98	18.05	23.45

The overall mean is $\bar{Y}_{..} = 22.245$
The parameter estimates are given here:
$\hat{\mu} = 22.245$, $\hat{\alpha}_1 = .255$, $\hat{\alpha}_2 = 2.735$ $\hat{\alpha}_3 = -4.195$, $\hat{\alpha}_4 = 1.205$
$\hat{\beta}_1 = .805$, $\hat{\beta}_2 = -1.165$, $\hat{\beta}_3 = .155$, $\beta_4 = .205$
$\hat{\gamma}_1 = -8.165$, $\hat{\gamma}_2 = 9.305$ $\hat{\gamma}_3 = -4.745$, $\hat{\alpha}_4 = 3.605$

c. The ANOVA table is given here:

Source	DF	SS	MS	F	p-value
Blend	3	106.272	35.424	8.22	0.015
Driver	3	8.332	2.777		
Car Model	3	755.372	251.791		
Error	6	25.864	4.311		
Total	15	895.839			

$p-value = 0.015 < 0.05 \Rightarrow$ Reject H_o and conclude there is significant evidence that there is a difference in the mean mileage of the four blends of gasoline.

d. The blend with the highest mileage is Blend B. However, there is very little difference between the sample means of Blends A, B, and D. In fact, the estimate $SE(\hat{\mu}_k) = \hat{\sigma}_\epsilon/\sqrt{n} = 4.311/\sqrt{4} = 2.16$. Therefore, Blends A, B, and D differ by only about one SE and hence would not be significantly different.

e. RE(LS,CR)$=\frac{MSR+MSC+(t-1)MSE}{(t+1)MSE} = \frac{2.777+251.791+(4-1)(4.311)}{((4+1)(4.311)} = 12.41 \Rightarrow$
It would take 12.41 times as many observations (approximately 50) per treatment in a completely randomized design to achieve the same level of precision in estimating the treatment means as was accomplished in the Latin square design.

f. In future studies, this type of study could be run as a RCB design using Car Model as the blocking variable because the four drivers had very little variation in their mileage.

14.8

a. Completely randomized design with a 3x2 factorial treatment structure and 10 reps.

b. $y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}; \quad i = 1,2,3; \quad j = 1,2; \quad k = 1,\cdots,10;$

where y_{ijk} is the attention span of the kth child of Age i viewing Product j.

α_i is the effect of the ith Age on attention span

β_j is the effect of the jth Product on attention span

$\alpha\beta_{ij}$ is the interaction effect of the ith Age and jth Product on attention span

c. The treatment means are given here:

A_1P_1	A_2P_1	A_3P_1	A_1P_2	A_2P_2	A_3P_2
22.9	19.6	21.9	23.1	30.5	45.6
A_1	A_2	A_3		P_1	P_2
23.0	25.05	33.75		21.47	33.07

$\hat{\mu} = 27.27, \quad \hat{\alpha}_1 = -4.27, \quad \hat{\alpha}_2 = -2.22, \quad \hat{\alpha}_3 = 6.48$

$\hat{\beta}_1 = -5.8, \quad \hat{\beta}_2 = 5.8$

$\hat{\alpha\beta}_{11} = 5.7, \quad \hat{\alpha\beta}_{21} = .35, \quad \hat{\alpha\beta}_{31} = -6.05$

$\hat{\alpha\beta}_{12} = -5.7, \quad \hat{\alpha\beta}_{22} = -.35, \quad \hat{\alpha\beta}_{32} = 6.05$

d. The ANOVA table is given here:

Source	DF	SS	MS	F	p-value
Age	2	1303.0	651.5	4.43	0.017
Product	1	2018.4	2018.4	13.72	0.001
Interaction	2	1384.3	692.1	4.70	0.013
Error	54	7944	147.1		
Total	59	12649.7			

prob. 4.

H_0: $(\alpha\beta)_{ij} = 0$ for all $i = 1, 2, 3$ & $j = 1, 2$.

H_a: At least one $(\alpha\beta)_{ij} \neq 0$.

$F = 4.70$ $df_1 = 2$, $df_2 = 54$.

$.01 < p\text{-value} < .025$

$\Rightarrow$ $p\text{-value} < \alpha = 0.05$ $\Rightarrow$ reject H_0.

There is significant evidence of an interaction between age & product type. That is, the effect of product type on the mean attention span of the children is different for children of different ages.

14.23

b. A model for this experiment is given here:

$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}; \quad i = 1,2,3,4; \quad j = 1,2,3; \quad k = 1,2,3;$

where y_{ijk} is the increase in trunk diameter of the kth tree in soil having the ith pH level using the jth Ca Rate:

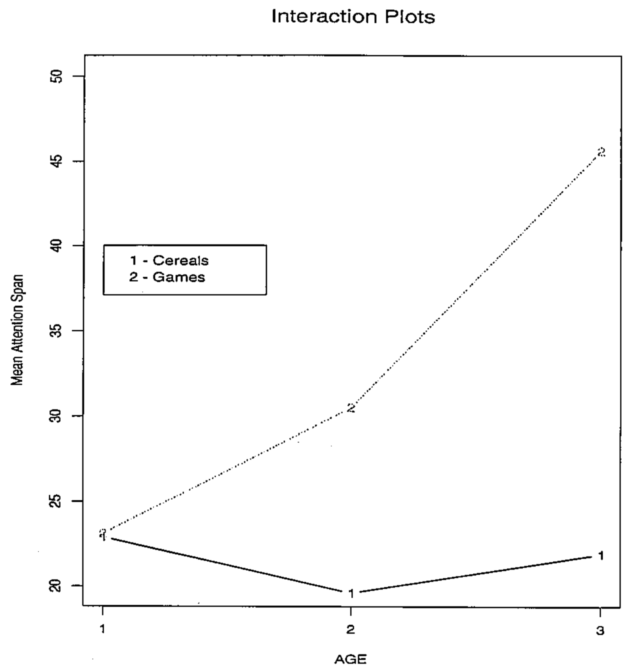
α_i is the effect of the ith pH level on diameter increase

β_j is the effect of the jth Ca Rate on diameter increase

$\alpha\beta_{ij}$ is the interaction effect of the ith pH level and jth Ca Rate on diameter increase.

c. This is a completely randomized 4x3 factorial experiment with Factor A: pH level, Factor B: Ca rate. There are 3 complete replications of the experiment.

14.9 a.



The profile plot indicates an increasing effect of Product Type as Age increases.

b. The p-value for the interaction term is 0.013. There is significant evidence of an interaction between the factors Age and Product Type. Thus, the size of the difference between mean attention span of children viewing breakfast cereals and viewing video games would be different for the three age groups. From the profile plots, the estimated mean attention span for video games is larger than for breakfast cereals, with the size of the difference becoming larger as age increases.

14.10 a. Similar results are obtained.

b. The residuals in the normal probability plot appear to fall very close to a straight line and hence we can conclude there is not evidence that the residuals have a non-normal distribution.

The plot of the residuals versus Fitted Value appear to have a consistent width across the fitted values. The condition of constant variance does not appear to be violated.