8.6

a. Yes, the mean for Device A is considerably (relative to the standard deviations) smaller than the mean for Device D.

b. $H_o : \mu_A = \mu_B = \mu_C = \mu_D$ versus H_a : Difference in $\mu' s$

Reject H_o if $F \geq F_{.05,3,20} = 3.10$

$SSW = 5[(.1767)^2 + (.2091)^2 + (.1532)^2 + (.2492)^2] = 0.8026$

$\bar{y}_{..} = 0.0826 \Rightarrow$

$SSB = 6[(-0.1605 - .0826)^2 + (0.0947 - .0826)^2 + (0.1227 - .0826)^2 + (0.2735 - .0826)^2]$

$= 0.5838 \Rightarrow$

$F = \frac{.5838/3}{.8026/20} = 4.85 > 3.10 \Rightarrow$

Reject H_o and conclude there is significant difference among the mean difference in pH readings for the four devices.

c. $p - value = P(F_{3,20} \geq 4.85) \Rightarrow p - value = 0.0107$

d. The data must be independently selected random samples from normal populations having the same value for σ.

Problem 3

a. The model for this experiment is given by

$y_{ij} = \mu + \alpha_i + \epsilon_{ij}; \quad i = 1, 2, 3 \;$ and $\; j = 1, \cdots, n_i$

where $n_1 = 6, n_2 = 5, n_3 = 4$; $\mu =$ overall mean; $\alpha_i =$ effect of ith nutrient;

$\epsilon_{ij} =$ random error associated with the jth response from the ith nutrient

b. $\mathrm{F} = \frac{317.08/2}{78.25/12} = 24.31$ with df = 2,12 $\Rightarrow p - value < 0.0001 < 0.05 \Rightarrow$

There is significant evidence of a difference in the mean starch content in the three nutrient groups.

4. Randomised Complete Block design

treatment: whether or not attended a Head Start program

block: student pair

Since the students are paired based on similarities in their home environment, the effect of home environment on academic performance is "adjusted for" in this experiment. Hence, any difference we may observe in students academic performance can be attributed to whether or not the student attended the Head Start program.

4

5. $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$

pair

Trt	1	2	3	4	5	6	mean
1	58	73	85	76	88	90	78.33
2	47	67	69	62	77	77	66.5
mean	52.5	70	77	69	82.5	83.5	72.42

$\hat{\mu} = \bar{y}_{..} = 72.42$

$\hat{\alpha}_i = \bar{y}_{i.} - \bar{y}_{..}$

$\hat{\alpha}_1 = 78.33 - 72.42 = 5.91$

$\hat{\alpha}_2 = 66.5 - 72.42 = -5.92$

$\hat{\beta}_j = \bar{y}_{.j} - \bar{y}_{..}$

$\hat{\beta}_1 = 52.5 - 72.42 = -19.92$

$\hat{\beta}_2 = 70 - 72.42 = -2.42$

$\hat{\beta}_3 = 77 - 72.42 = 4.58$

$\hat{\beta}_4 = 69 - 72.42 = -3.42$

$\hat{\beta}_5 = 82.5 - 72.42 = 10.08$

$\hat{\beta}_6 = 83.5 - 72.42 = 11.08$

problem 6.

a. The F-test from the ANOVA table tests 2-sided alternatives:
Test $H_o : \mu_{Attend} = \mu_{DidNot}$ vs. $H_o : \mu_{Attend} \neq \mu_{DidNot}$
The ANOVA table is given here:

Source	DF	SS	MS	F	p-value
Pair	5	1319.42	263.88		
Treatment	1	420.08	420.08	71.40	0.0004
Error	5	29.42	5.88		
Total	11	1768.92			

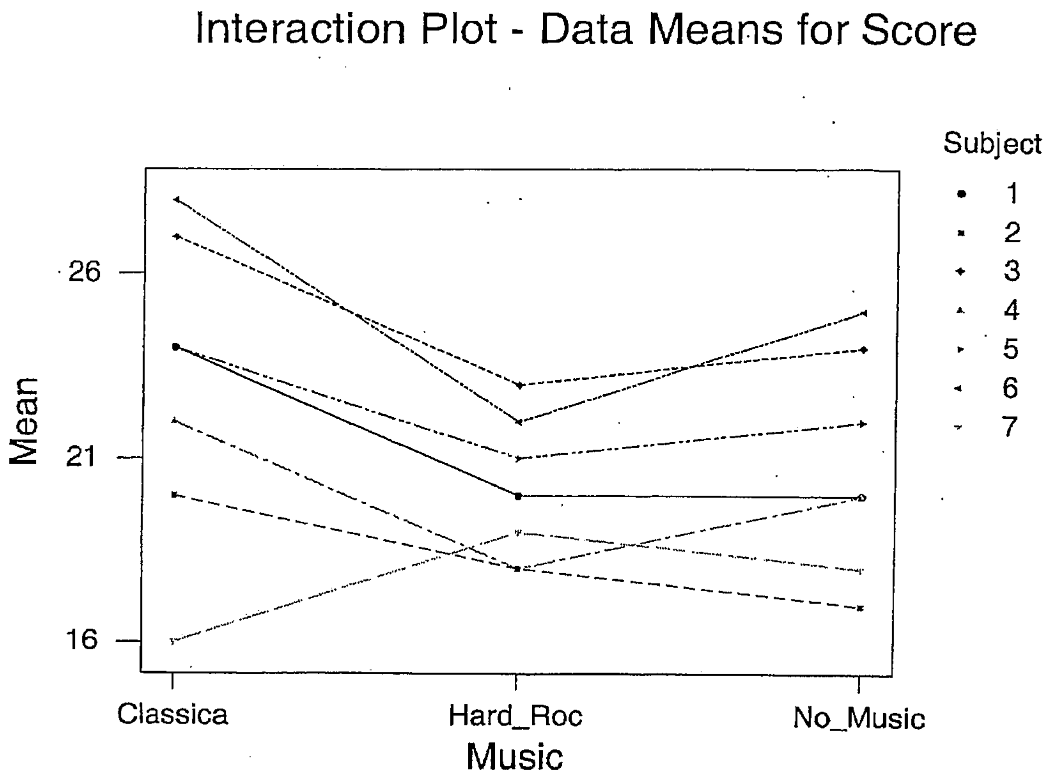
Reject H_o and conclude there is significant evidence that the mean scores of students attending Head Start are significantly different from the mean scores of students who do not attend Head Start.

b. RE(RCB,CR)=$\frac{(b-1)MSB+b(t-1)MSE}{(bt-1)MSE} = \frac{(6-1)(263.88)+(6)(2-1)(5.88)}{((6)(2)-1)(5.88)} = 20.94 \Rightarrow$
It would take approximately 21 times as many observations (126) per treatment in a completely randomized design to achieve the same level of precision in estimating the treatment means as was accomplished in the randomized complete block design.

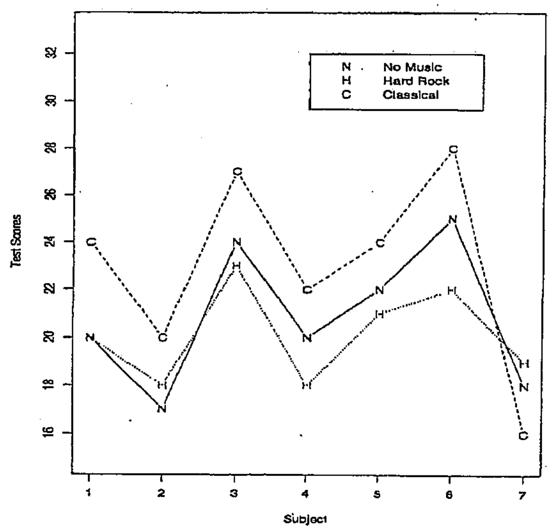
15.16 a. $y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$; $i = 1, 2, 3$, $j = 1, 2, 3, 4, 5, 6, 7$
y_{ij} is score on test of jth subject hearing the ith music type
α_i is the ith music type effect
β_j is the jth subject effect
$\hat{\mu} = 21.33$, $\hat{\alpha}_1 = -0.47$, $\hat{\alpha}_2 = -1.19$, $\hat{\alpha}_3 = 1.67$
$\hat{\beta}_1 = 0$, $\hat{\beta}_2 = -3$, $\hat{\beta}_3 = 3.33$, $\hat{\beta}_4 = -1.33$, $\hat{\beta}_5 = 1$, $\hat{\beta}_6 = 3.67$, $\hat{\beta}_7 = -3.67$

b. $F = \frac{SS_{TRT}/df_{TRT}}{SS_{Error}/df_{Error}} = \frac{30.952/2}{28.38/12} = 6.54$ with df=2,12.
Therefore, $p - value = Pr(F_{2,12} \geq 6.54) = 0.0120 \Rightarrow$ Reject $H_o : \mu_1 = \mu_2 = \mu_3$.
We thus conclude that there is significant evidence of a difference in mean typing scores for the three types of music.

c. An interaction plot of the data is given here:



OR:



Based on the interaction plot, the additive model may be inappropriate because there is some crossing of the three lines. However, the plotted points are means of a single observation and hence may be quite variable in their estimation of the population means μ_{ij}. Thus, exact parallelism is not required in the data plots to ensure the validity of the additive model.

d. $t = 3, b = 7 \Rightarrow RE(RCB, CR) = \frac{(7-1)(24.889)+(7)(3-1)(2.365)}{((7)(3)-1)(2.365)} = 3.86 \Rightarrow$

It would take 3.86 times as many observations (approximately 27) per treatment in a completely randomized design to achieve the same level of precision in estimating the treatment means as was accomplished in the randomized complete block design. Since RE was much larger than 1, we would conclude that the blocking was effective.