1. (a) $S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 17411 - \frac{(507)^2}{16} = 1345.437$

$S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} = 49443.83 - \frac{(507)(1563.47)}{16}$

≈ -98.626

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{-98.626}{1345.437} \approx -0.0733$

$\bar{y} = \frac{\sum y}{n} = \frac{1563.47}{16} = 97.717$

$\bar{x} = \frac{\sum x}{n} = \frac{507}{16} = 31.688$

$b_0 = \bar{y} - b_1\bar{x} = 97.717 - (-0.0733)\,31.688$

$= 100.04$

Hence, the LS regression line is $\hat{y} = 100.04 - 0.073x$

(b) $\hat{y} = 100.04 - 0.073(38) = 97.27$

residual $= y - \hat{y} = 111.49 - 97.27 = 14.22$

(c) $S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} = 154571.5 - \frac{(1563.47)^2}{16}$

$= 1794.097$

$S_\varepsilon = \sqrt{\frac{S_{yy} - S_{xy}^2/S_{xx}}{n-2}}$

$= \sqrt{\frac{1794.097 - (-98.626)^2/1345.437}{16-2}} \approx 11.297$

$SE_{\hat{\beta}_1} = \frac{S_\varepsilon}{\sqrt{S_{xx}}} = \frac{11.297}{\sqrt{1345.437}} \approx 0.308$

95% C.I. for β_1:

$\hat{\beta}_1 \pm t(n-2)_{\alpha/2}\, SE_{\hat{\beta}_1}$

1(c) (continued)

$= -0.0733 \pm 2.145(0.308)$

$= -0.073 \pm 0.66 = (-0.733, 0.587)$

We are 95% confident that the change in the bone density is between -0.733 and 0.587 per year increase of age.

(d) Since the 95% C.I. for β_1 contains 0, there is no linear relationship between bone density and age based on these data.

2. (a) $H_0: \beta_1 = 0$ vs. $H_a: \beta_1 \neq 0$

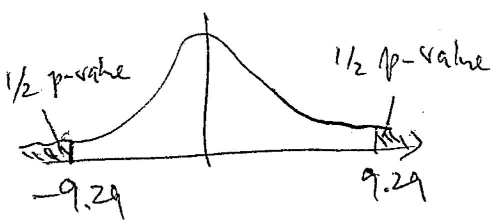
$$t = \frac{\hat{\beta}_1 - 0}{SE_{\hat{\beta}_1}} = \frac{3.2212}{0.3467} = 9.29, \quad df = n-2 = 21$$

R.R. Reject H_0 if $t \geq t_{\alpha/2}(df)$

Since $9.29 > t_{.025}(21) = 2.080 \Rightarrow$ reject H_0.

OR: $p\text{-value} = 2P(t(21) \geq 9.29) < .0005 \times 2 < \alpha = 0.05$

$\Rightarrow$ reject H_0



Yes, there is a linear relationship between assessed valuation and selling price.

(b) $r = \text{sign}(\hat{\beta}_1)\sqrt{r^2}$

$$= +\sqrt{\frac{SSR}{SST}} = \sqrt{\frac{2279.2 - 445.9}{2279.2}} = 0.897$$

There is a strong positive linear relationship between assessed valuation and selling price.

(c) 99% C.I. for EY_{n+1}: (40.363, 45.805)

We are 99% confident that the mean selling price for many houses with assessed valuation of \$14.5K will be between \$40.363K and \$45.805K.

99% P.I. for Y_{n+1}: (29.756, 56.412)

We are 99% confident that the selling price for a single house with an assessed valuation of \$14.5K will be between \$29.756K and \$56.412K.

2(c) (continued)

Based on these data, it seems that the assessed values are too low compared to selling prices.

(d) $\hat{y} = -2.37 + 2.92\,\text{Assessed} + 7.82\,\text{Corner}$

The average selling price of a corner house is higher than that of a non-corner house by \$7.82k, when comparing houses with the same assessed value.

(e) $H_0: \beta_1 = \beta_2 = 0$ vs. H_a: At least one of $\beta_1, \beta_2 \neq 0$

$$F = \frac{SSR/2}{SSE/(n-2-1)} = \frac{2152.2/2}{127/20}$$

$$= \frac{1076.1}{6.35} = 169.46$$

$df_1 = 2$, $df_2 = 20$

RR: Reject H_0 if $F \geq F_\alpha(df_1, df_2)$

Since $169.46 > F_{0.05}(2, 20) = 3.49$

$\Rightarrow$ reject H_0.

OR: $p\text{-value} = P(F(2,20) \geq 169.46) < .001 < \alpha$

$\Rightarrow$ reject H_0.

Yes, assessed valuation and corner house indicator together have statistically significant predictive value for predicting selling price.

2. (f) $SSR(x_2|x_1) = SSR(x_1, x_2) - SSR(x_1)$

$= 2152.2 - (2279.2 - 445.9)$

$= 318.9$

This is the extra variation in y that is explained by adding x_2 (corner) to the model that already has x_1 (assessed).

(g) Complete model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$

Reduced model: $y = \beta_0 + \beta_1 x_1 + \varepsilon$

H_0: $\beta_2 = 0$

H_a: $\beta_2 \neq 0$.