4.82

a. $P(y > 7) = P\left(z > \frac{7-5}{1.3}\right) = P(z > 1.54) = 0.0618$

b. $P(\bar{y} > 5.5) = P\left(z > \frac{5.5-5}{1.3/\sqrt{500}}\right) = P(z > 8.6) \approx 0$ The results of the survey are not consistent.

4.85 Individual baggage weight has $\mu = 95$ and $\sigma = 35$. Total weight has mean $n\mu = 200(95) = 19,000$ and standard deviation $\sqrt{n}\sigma = \sqrt{200}(35) = 494.97$. Therefore, $P(y > 20,000) = P\left(z > \frac{20,000-19,000}{494.97}\right) = P(z > 2.02) = 0.0217$.

4.86

a. $P(y \le 150) = P\left(z \le \frac{150-160}{20}\right) = P(z < -0.5) = 0.3085$

b. $P(\bar{y} \le 150) = P\left(z \le \frac{150-160}{20/\sqrt{5}}\right) = P(z < -1.12) = 0.1314$

c. $P(\bar{y} \le 150) = P\left(z \le \frac{150-160}{20/\sqrt{n}}\right) = P(z \le -2.326) = 0.01 \Rightarrow \frac{150-160}{20/\sqrt{n}} = -2.326 \Rightarrow n = 21.64$

At least 22 measurements would be needed.

4.88 For the binomial distribution, $P(y < 5) = P(y \le 4)$. Thus, we have

a. $P(y < 5) = P(y \le 4) = \sum_{i=0}^{4} \binom{20}{i} (0.5)^i (0.5)^{20-i} = 0.0059$

$\mu = n\pi = (20)(0.5) = 10$; $\sigma = \sqrt{(20)(0.5)(0.5)} = 2.236$;

$P(y < 5) = P(y \le 4) \approx P\left(z \le \frac{4-10}{2.236}\right) = P(z \le -2.68) = 0.0037$

b. $P(y < 5) = P(y \le 4) \approx P\left(z \le \frac{4.5-10}{2.236}\right) = P(z \le -2.46) = 0.0069$

c. $P(8 < y < 14) = P(9 \le y \le 13) = \sum_{i=9}^{13} \binom{20}{i} (0.5)^i (0.5)^{20-i} = 0.6906$

Using the normal approximation with correction:

$P(8 < y < 14) = P(9 \le y \le 13) = P(y \le 13) - P(y \le 8)$

$\approx P\left(z \le \frac{13.5-10}{2.236}\right) - P\left(z \le \frac{8.5-10}{2.236}\right) = P(z \le 1.57) - P(z \le -0.67) = 0.6904$

4.102

a. $\mu = n\pi = (400)(0.20) = 80$; $\sigma = \sqrt{(400)(0.20)(0.80)} = 8$;

$$P(y \le 25) \approx P\left(z \le \frac{25.5 - 80}{8}\right) = P(z \le -6.81) \approx 0$$

b. The ad is not successful. With $\pi = 0.20$, we expect 80 positive responses out of 400 but we observed only 25. The probability of getting so few positive responses is virtually 0 if $\pi = 0.20$. We therefore conclude that π is much less than 0.20.

5.5

a. $110 \pm (1.96)\frac{7.1}{\sqrt{50}} = 110 \pm 1.97 = (108.0, 112.0)$

b. We are 95% confident that the average caffeine content is between 108 and 112 milligrams.

5.9 $3.2 \pm (1.96)\frac{1.1}{\sqrt{150}} = 3.2 \pm 0.18 = (3.02, 3.38)$

5.63

a. $\bar{y} = 28.7$

b. $\frac{s}{\sqrt{n}} = \frac{3.8}{\sqrt{38}} = 0.62$

c. 95% CI: $28.7 \pm 1.96\frac{3.8}{\sqrt{38}} = 28.7 \pm 1.21 = (27.49, 29.91)$ We are 95% confident that the average time to handle a customer complaint is between 27.49 and 29.91 minutes.

d. Test $H_0 : \mu \ge 30$ versus $H_a : \mu < 30$.

$$z = \frac{28.7 - 30}{3.8 / \sqrt{38}} = -2.11 \Rightarrow \text{ p-value} = P(z < -2.11) = 0.0174 < 0.05 = \alpha$$

$\Rightarrow$ Reject H_0. There is sufficient evidence to conclude that the mean time to handle a customer complaint has decreased.