Solution of Homework 2

Q: 2.3.2 Since \((A \cap B^C) \cup (A^C \cap B) \iff A \text{ or } B \text{ not both occur}\), we have

\[
P[(A \cap B^C) \cup (A^C \cap B)] = P(A \cap B^C) + P(A^C \cap B)
= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]
= 0.4 - 0.1 + 0.5 - 0.1
= 0.7
\]

Q: 2.3.3

(a) Since \(A^C \cup B^C = (A \cap B)^C\), by using the property of probability, we have

\[
P(A^C \cup B^C) = 1 - P(A \cap B).
\]

(b) \(P[A^C \cap (A \cup B)] = P[(A^C \cap A) \cup (A^C \cap B)] = P(B) - P(A \cap B)\).

Q: 2.3.10 Let events \(A\) and \(B\) be \(A = \{\text{the number is divisible by 2}\} = \{2, 4, 6...24, \}\)
and \(B = \{3, 6, 9, 12, 15, 18, 21, 24\}\). Hence,

\[
P(A \cup B) = \frac{16}{24} = \frac{2}{3}
\]

Q: 2.3.11 Let \(A = \{\text{state’s football team wins Saturday}\}, \) and \(B = \{\text{state’s football team wins next Saturday}\}. \) Then \(P(A) = 0.1, P(B) = 0.3. \) It follows that

\[
P(\text{lose both}) = P(A^C \cap B^C) = 1 - P(A \cup B) = 0.65
\]

\[
P(\text{wins exactly once}) = P[(A^C \cap B^C) \cup (A^C \cap B)]
= P(A \cup B) - P(A \cap B)
= 0.35 - 0.05 = 0.3
\]

Q: 2.3.12 Since \(A_1 \cup A_2 = S, A_1 \cap A_2 = 0, \) the events \(A_1 \) and \(A_2 \) is partition of \(S.\)

\[
P(A_1 \cup A_2) = P(A_1) + P(A_2) = 1, P(A_1) = 1 - P(A_2),
\]
i.e., \( p_1 = 1 - P_2 \). Then \( p_1 \) and \( p_2 \) satisfy the following equation:

\[
\begin{aligned}
& p_1 = 1 - p_2 \\
& 3p_1 - p_2 = \frac{1}{2}
\end{aligned}
\]

By solving the above equation, the solution \( p_2 = \frac{5}{8} \).

**Q: 2.3.16** Let the events \( A = \{ \text{The sum of faces is 6} \} \) and \( B = \{ \text{The face showing on one die is twice the face showing on the other} \} \). So we have

\[
\begin{align*}
A &= \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\} \\
B &= \{(1, 2), (2, 4), (3, 6), (2, 1), (4, 2), (6, 3)\}
\end{align*}
\]

Therefore, \( P(A) = \frac{5}{36} \), \( P(B) = \frac{6}{36} \), and

\[
P(A \cap B^C) = P(A) - P(A \cap B) = \frac{5}{36} - \frac{2}{36} = \frac{3}{36}
\]