

MATH 3C PRACTICE MIDTERM

Lecture 2, W'03, Feb 3, 2003

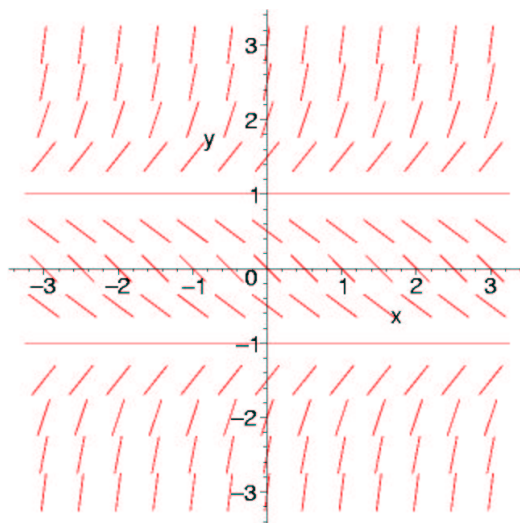
All of your answers must be carefully justified. It is not enough to have a correct answer, you must explain how you got it. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may not use books, notes, and calculators on this exam.

Please, bring scratch paper. Pencil and eraser are recommended tools, and a ruler may come in handy. You may also have a $3'' \times 5''$ handwritten cheat sheet (both sides). The exam is based on all the material covered through lecture on 2/3 up to and including FHMW 2.1.

1. Decide if the following statements are true (i.e. always true) or false (i.e. false in at least one case). You need not justify your answer.
 - (a) An autonomous first-order DE is of the form $y' = f(y)$ where f is some function.
 - (b) The DE $y'' - x^2y' + 2xy - \sin(y) = 0$ is linear.
 - (c) The DE $y' + \frac{y}{x^2} + 5x = 0$ is homogeneous.
 - (d) $y(x) = \sin x + \cos x$ is a solution of the DE $y'' + y = 0$.
 - (e) If $y_1(x)$ and $y_2(x)$ are solutions of the DE $y'' - e^xy + \sin(x) = 0$, then $y_1(x) + y_2(x)$ is also a solution of this DE.
 - (f) $L(y) = \frac{d^2y}{dx^2} + 2(y+x)\frac{dy}{dx}$ is a linear differential operator.
 - (g) If y_n is a nonzero solution of the DE $y' + \sin(x)y = x^2$ and y_h is a nonzero solution of the DE $y' + \sin(x)y = 0$, then $y = y_n + Cy_h$ is a solution of $y' + \sin(x)y = x^2$ for any number C .
 - (h) The DE $\frac{y'}{x^2} = e^{x+y^2}$ is separable.
 - (i) Suppose that $y(x) = c$ is an equilibrium solution of the DE $y' = f(x, y)$. Then $f(x, c) = 0$ for all numbers x .
 - (j) The isoclines of an autonomous first-order DE cannot intersect each other.

2. Find the equation of the tangent plane to the function $f(x, y) = x \ln(x - y^2) + y$ at the point $(5, 2)$.

3. The plot to the right shows the slope field of the first-order differential equation $y' = f(x, y)$.



- (a) Sketch some isoclines. How can you tell from the slope field if the equation is autonomous?
- (b) Find a differential equation that fits this slope field.
- (c) Sketch a solution with the initial value $y(1) = 0$.
- (d) Can you find any equilibrium solutions?

4. Let

$$\frac{dy}{dx} - 2xy = \frac{1}{x} \quad (\text{where } x \neq 0).$$

- (a) Verify that $y(x) = \int_1^x \frac{e^{x^2-t^2}}{t} dt$ is a solution. (Recall the FTC: $\frac{d}{dx} \int_c^x f(t) dt = f(x)$.)
- (b) Find the general solution. (Hint: solve the corresponding homogeneous equation.)
- (c) Check that your solution is correct by substituting it back into the original equation.
- (d) Find the particular solution that fits the initial condition $y(0) = 2$.