Name: ______________________ Section (1 pt): ☐ 8AM, ☐ 5PM, ☐ 6PM, ☐ 7PM, ☐ MAP

Perm # (1 pt): ______________

Math 3C Midterm

Lecture 2, W'03, Feb 10, 2003

All your answers must be carefully justified. It is not enough to have a correct answer, you must explain how you got it. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may not use books, notes, and calculators on this exam. You may have a $3'' \times 5''$ handwritten cheat sheet.

Simplify your answers as much as you can. It's ok to leave $\sqrt{3}$, π and such, but not 7/14. Put your answers on this sheet. You may use the back of the sheets if you need more space. This exam has 2 pages.

0	2
1	
2	
3	
4	
$\sum$	

For office use only. Don't put anything in these boxes.

1. (2 pts for each correct answer, -1 pt for each wrong answer, 0 for blank) Decide if the following statements are true (i.e. always true) or false (i.e. false in at least one case). You need not justify your answer.
   (a) ☐ Every autonomous first-order DE is separable.
   (b) ☐ The DE $e^{x^2}y' + e^{x+y} = 0$ is linear.
   (c) ☐ The DE $x^2y'' + \frac{y'}{\cos(x)} = 0$ is a linear and homogeneous.
   (d) ☐ If L is a linear operator and y_1 and y_2 are differentiable functions, then $L(y_1)+L(y_2) = L(y_1 + y_2)$.
   (e) ☐ If L is an operator and $L(y_1 + y_2) = L(y_1) + L(y_2)$ for any differentiable functions y_1 and y_2, then L is a linear operator.
   (f) ☐ The isoclines of the DE $y' = f(x, y)$ are the level curves of the function $f(x, y)$.
   (g) ☐ The DE $e^xy' + e^{x+y} = e^{2x}$ is separable.
   (h) ☐ If y is an equilibrium solution of a first-order DE, then $y'(x) = 0$ for all x.
   (i) ☐ Let L be a linear differential operator and y a solution of the DE $L(y) = \sin^2(x)$. Then cy is also a solution of the same DE for any constant c.
   (j) ☐ The DE $y' - (x^3 - 1)e^y = 0$ has no equilibrium solutions.

2. (10 pts) Find the equation of the tangent plane to the function $f(x, y) = e^{x^2} + x^y$ at the point $(1, 3)$.

3. (10 pts) In this problem, you will use Euler's method with a stepsize of $\Delta x = 0.5$ to approximate the solution of the IVP

$$\frac{dy}{dx} = 2x - y, \qquad y(2) = 1.$$

Fill out the table below:

x	y	$2x - y$	Δy
2			
2.5			
3			

4. (15 pts) In this problem, you will find solutions of the DE

$$\frac{dy}{dx} = \frac{(y^3 - y)2x \ln x}{3y^2 - 1} \qquad (x > 0).$$

(a) Find the equilibrium solutions if there are any.

(b) Use separation of variables to find all the nonequilibrium solutions. You may not be able to solve for y explicitly.

(c) Check your solution by differentiating it. (Hint: you may have to use implicit differentiation.)

(d) Find the particular solution that fits the initial condition $y(1) = 2$.