

Math 2111 Practice Problems

We will do these problems together in class on Fri (Oct 14). Please, look attempt them beforehand. You will get more out of the discussion that way, not to mention that you learn a lot more from doing problems than from listening to someone else presenting solutions.

1. Do 3.5.25, 26, 41, 42 in the book.
2. Let V be an n -dimensional vector space and S a subset of $m < n$ elements. Prove that S cannot be a spanning set.
3. Compute the following dimensions and justify your answer.
 - (a) $\dim_{\mathbb{C}}(\mathbb{C})$
 - (b) $\dim_{\mathbb{R}}(\mathbb{C})$
4. What do you think $\dim_{\mathbb{Q}}(\mathbb{R})$ is? (Hint: \mathbb{Q} is countable, \mathbb{R} is not.) Can you think of a way to justify your answer? If you can't, it's ok, this is probably a little hard at this point.
5. Do 3.5.19-22 in the book.
6. Let V be a finite dimensional vector space. We proved in class that if $S \subseteq V$ is a spanning set and $T \subseteq V$ is linearly independent, then $|T| \leq |S|$.

Now, let $V = \mathbb{R}^4$ and

$$S = \{(1, 0, 1, 0), (1, 0, -1, 0), (-1, 2, 2, 1), (1, 0, 2, -1), (0, 2, 3, 1)\}$$

$$T = \{(1, 2, 1, 1), (1, 1, 2, 1), (0, 1, 0, -1)\}$$

Verify that S spans V and T is linearly independent. Now, recall how the proof of the above theorem went and use the same argument in this specific case to prove that $|T| \leq |S|$.