Quantification
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# 1 Generalized Quantifiers

- Most

(1) a. Most bunnies eat twinkies.
b. The number of twinkie-eating bunnies is (somewhat/much) greater than the number of non-twinkie eating bunnies.
c. Wrong: Most $x[\text{bunny}(x) \rightarrow \text{eat}(x, \text{twinkies})]$
d. Wrong: Most $x[\text{bunny}(x) \wedge \text{eat}(x, \text{twinkies})]$
e. Wrong: Most $x[\text{bunny}(x) \vee \text{eat}(x, \text{twinkies})]$

- Notation:

$[\![\text{bunny}]\!]$ = the set of bunnies
$\mid [\![\text{bunny}]\!] \mid =$ the number of bunnies
the **cardinality** of the set of bunnies
$\{x \mid \text{eat}(x, \text{twinkies})\}$ = the set of twinkie-eaters
$\{x \mid \text{bunny}(x)\}$ = another way of writing the set of bunnies
$\emptyset$ = the set with no members; the empty set, which is unique

- The right truth conditions

$\frac{|\{x|\text{bunny}(x) \wedge \text{eat}(x,\text{twinkies})\}|}{|\{x|\text{bunny}(x)\}|} > \frac{1}{2}$
the number of twinkie eating bunnies divided by the number of bunnies is greater than $\frac{1}{2}$

Note: This ain't first order logic. We can't talk about properties of sets in first-order logic (properties such as their cardinality)

- Look ma. I'm doing higher-order logic (Second order).
- First-order logic cannot describe the truth-conditions of *most*: Provable
- No big deal. Our goal: The truth conditions of English [Not translatability into any particular representation]

- Quantifiers are relations between SETS
  - Asymmetric quantifiers
    - Every boy walks: The set of boys is a **subset** of the set of walkers

      $$\{x \mid \text{boy}(x)\} \subseteq \{x \mid \text{walk}(x)\}$$

    - No boy walks: The intersection of the set of boys with the set of walkers is empty

      $$\{x \mid \text{boy}(x)\} \cap \{x \mid \text{walk}(x)\} = \emptyset$$

    - Most boys walk: The ratio of the size of the intersection of the set of boys with the set of walkers to the size of the set of boys is greater than $\frac{1}{2}$:

      $$\frac{\mid \{x \mid \text{boy}(x)\} \cap \{x \mid \text{walk}(x)\} \mid}{\mid \{x \mid \text{boy}(x)\} \mid} \geq \frac{1}{2}$$

  - Cardinality Quantifiers (Symmetric)
    - 6 boys walk: The size of the intersection of the set of boys with the set of walkers is 6

      $$\mid \{x \mid \text{boy}(x)\} \cap \{x \mid \text{walk}(x)\} \mid = 6$$

    - Many boys walk: The size of the intersection of the set of boys with the set of walkers is many

      $$\mid \{x \mid \text{boy}(x)\} \cap \{x \mid \text{walk}(x)\} \mid = \text{many}$$

      Or maybe better:

      $$\text{many}(\mid \{x \mid \text{boy}(x)\} \cap \{x \mid \text{walk}(x)\} \mid)$$

    - Several boys walk: The size of the intersection of the set of boys with the set of walkers is several

      $$\mid \{x \mid \text{boy}(x)\} \cap \{x \mid \text{walk}(x)\} \mid = \text{several}$$

- For symmetric quantifiers the following entailment pattern holds:

  Quant P Q → Quant Q P
  Many boys smoke → Many smokers are boys
  7 boys smoke → 7 smokers are boys

- For asymmetric quantifier this entailment pattern fails:

  Quant P Q ↛ Quant Q P
  All Lithuanians are smart ↛ All smart people are Lithuanians

  Quant P Q ↛ Quant Q P
  Most Lithuanians are smart ↛ Most smart people are Lithuanians

## 2 Few and Many

- Strong and weak readings: Asymmetric and symmetric readings
  - Strong: Proportionately there were few

    (2) a. Many fleas were tested. Few fleas survived.
    b. 89 out of 1000 tested fleas survived: true
    c. 89 out of 100 tested fleas survived: false
    d. Often the context supplies a set of fleas allowing us to measure the proportion of that set having a certain property
    e. Often paraphrasable with *few of the X*: Many fleas were tested. Few of the fleas survived.

  - Weak: No proportion, a cardinality quantifer, some number counts as few

    (3) a. The house seemed clean and Lee found (very) few fleas.
    b. 89: false
    c. 8: true

  - Familiarity: A set or entity is **familiar** in the discourse, when it has been mentioned before.
  - Strong NPs can make two kinds of claims:
    - Claims about a familar set

* General claims

This exhausts the possibilities. A strong NP can't be used to introduce a previously undefined set into the discourse, because it makes proportionality claims. We need to know the identity of two sets in order to evaluate the claims. [Most guns smoke: set of guns and set of smoking guns.]

**Strong NPs:** Illustrating familiarity effects

(4) a. All men must report before leaving. [All men *in the battalion*]
  b. Most people voted for Continuance.. [Most people *voting in the election we've been discussing*; note usually not even *most among the eligible voters*]
  c. Few cars are expected to finish the trial.[Few cars *in the race*]

**Strong NPs**: General claims

(5) a. All men are mortal.
  b. Most people are protective of children.
  c. Few cars can exceed 180 m.p.h.

**Weak NPs:** Novelty Possible

(6) a. The house seemed clean and Lee found very few fleas.

- Not a general claim about all fleas and not a claim about a previously introduced set of fleas.
- Introduces a novel set of fleas (the set Lee found)

(7) $\{x \mid \text{flea}(x) \wedge \text{find}(\text{lee},\ x)\}$

- Asserts *fewness* of them
- Possible with a cardinality quantifier, because the claim is not about the proportion-properties of two sets but about a cardinality claim about one set.
- You can't assert *mostness* of the set in (7 )

(8) # The house seemed clean and Lee found most fleas.

- Why can't you assert mostness: a claim involving mostness requires a comparison between TWO sets to be coherent

– Summarizing weak and strong: Quantifiers with weak readings are those with a cardinality interpretation:

| Quantifier | Weak? | Strong? |
|---|---|---|
| Every | No | Yes |
| Most | No | Yes |
| Seven | Yes | No |
| Some | Yes | No |

– TREMENDOUSLY confusing: the term **strong reading** is used to refer both to the only readings you get with strong NPs, and to the reading you get with weak quantifiers when they exploit familiar sets (sometimes called *partitive readings*: (Buring)

(9) a. There were 117 cowboys at the Halloween hoedown.
b. Many cowboys were bitterly disappointed when Patsy Cline failed to appear.
c. [= Many OF THE COWBOYS]decided to go home.
d. Some cowboys decided to go home.
e. [= Some OF THE COWBOYS ] decided to go home.

# 3 Existential There BE (* Strong Quantifiers)

– This construction simply asserts existence: Common discourse function: Introduction of *novel* participant (opposite of familiar) into the dialogue. Hence sometimes called **Presentational Construction**

(10) There is a Santa Clause. He lives at the North Pole and chews breath mints.

– There BE construction has constraints on what kinds of NPs it hosts: **No strong NPs**

(11) a. There was a dog in the garden. [presentational sentence/there BE sentence]
b. There were several dogs in the garden.
c. There were many dogs in the garden.
d. There were four dogs in the garden.

e. # There was every dog in the garden. [No strong quantifiers]
f. # There were most guests in the garden.

– No definites either

(12) a. Proper Name: # There was Terry in the garden. [#: There BE reading: OK: Inverted Locative, compare *There stood Terry in the garden.*, *There Terry was in the garden.*]
b. Demonstrative: # There was that dog in the garden.
c. Pronoun: # There were they/them in the garden.
d. Possessive: # There was Terry's dog in the garden.

– Potential Counterexample to no definites claim

(13) a. [Speaker A] I don't think we saw a single unmarried woman at the party.
b. [Speaker B] There was that widow from Sedona. [appears to be an existential claim; not an inverted locative; but serves a different discourse function from introduction of unfamiliar entity into the dialogue.]

# 4 Quantifier scope ambiguities

- Ambiguous

(14) a. Everyone in this room speaks two languages.
b. Wide scope for *two languages*: There are two languages everyone in this room speaks (namely Finnish and Hungarian)
c. Wide scope forced: Everyone in this room speaks two languages, namely Finnish and Hungarian.
d. Wide scope for *Everyone*: Everyone in this room is bilingual.

- Notation

| Wide Scope | Formula |
|---|---|
| Everyone in this room | $[\forall x \text{in}(x, \text{this room})]$ $[\text{Two } y \text{ language}(y)]$ $\text{speak}(x, y)$ |
| Two languages | $[\text{Two } y \text{ language}(y)]$ $[\forall x \text{in}(x, \text{this room})]$ $\text{speak}(x, y)$ |

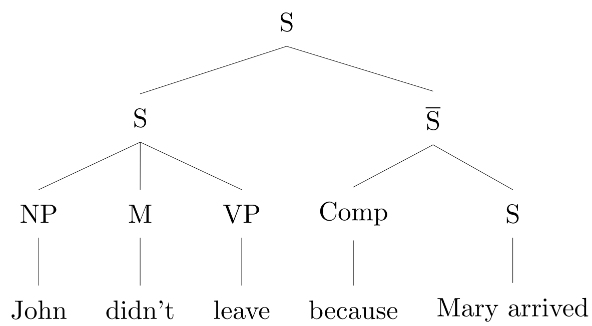


Figure 1: Surface structure tree for *John didn't leave because Susan came.* Wide scope *because*

- Scope ambiguities can be constrained by surface structure (Figures 1 and 2)
- But sometimes they are accounted for only in Logical Form [LF] (Figures 3 and 4)
- Distinguishing the truth conditions of some scopings takes real thought; see figure 5 for Sentence 15

(15) a. Most boys like most girls.
b. Most girls are liked by most boys.

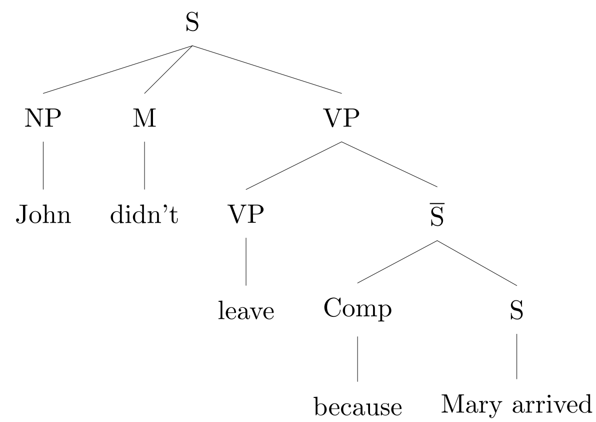


Figure 2: Surface structure tree for *John didn't leave because Susan came.* Wide scope *not*

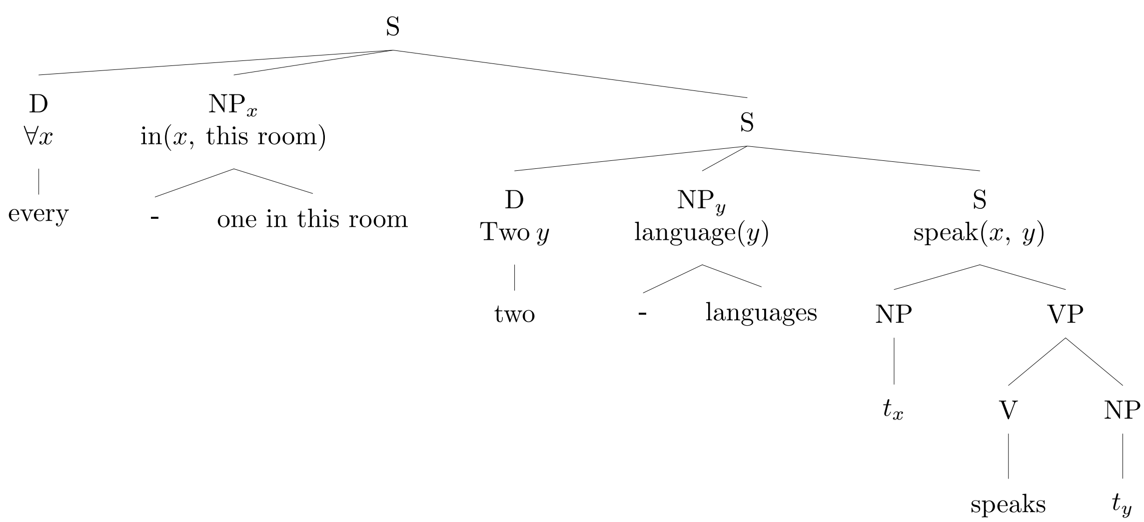


Figure 3: Logical Form tree for *Everyone in this room speaks two languages.* Wide scope *Every.*

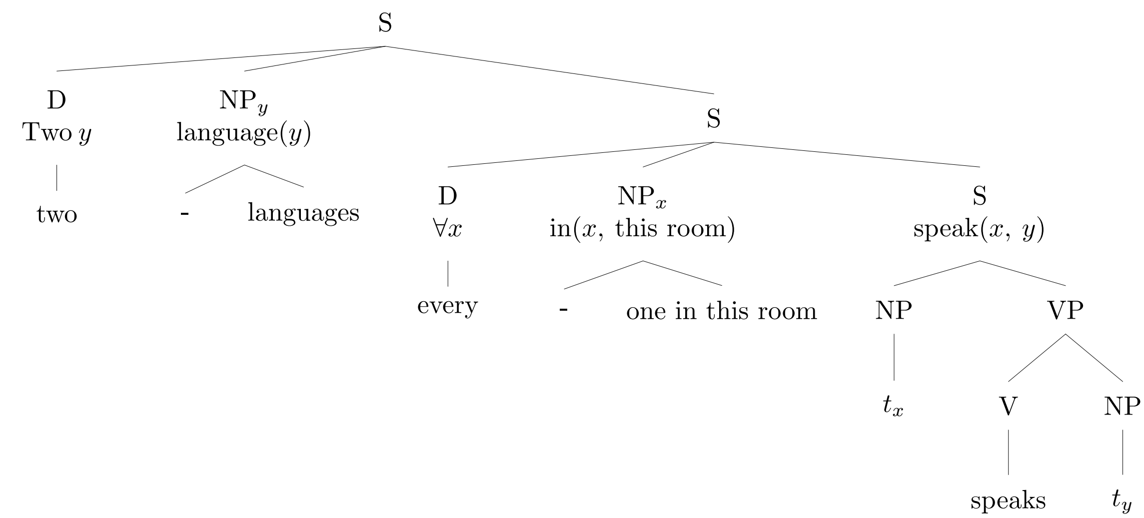


Figure 4: Second Logical Form tree for *Everyone in this room speaks two languages.* Wide scope *Two.*

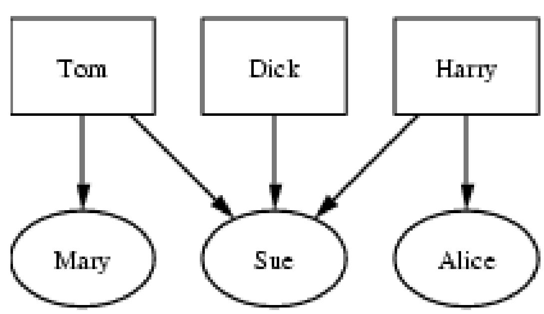


Figure 5: A situation in which (15a) is true and (15b) is not