Logical Translations

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1 Introduction

We introduce the connectives of first-order logic.

We introduce predicates. And a very simple semantics for them.

2 Truth-Functional Connectives

2.1 And

	p	q	$p \wedge q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	F
(d)	F	F	F

Only example (a) really works as if $\wedge$ in the above truth-table was the right translation for *and*

(1)
a. Abraham Lincoln was elected in 1860 and he was re-elected in 1864.
b. John picked up the apple and he ate it.
c. ? John ate the apple and he picked it up. [temporal order]
d. You take one more step and I'll shoot. [= If you take one more step, I'll shoot]

2.2 Or

	p	q	$p \vee q$
(a)	T	T	T
(b)	T	F	T
(c)	F	T	T
(d)	F	F	F

(2) a. He rented either a mid-size or an economy car. [If in fact he rented both, this is still true]
b. Either there's no bathroom in this house or it's on the second floor. [In fact both statement can't be simultaneously true, but that's not due to the meaning of *or*]
c. You can have either the white one or the red one. [intended meaning: but not both]

2.3 material implication

Material implication is the name we'll use for $\rightarrow$.

	p	q	$p \rightarrow q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	T
(d)	F	F	T

(3) a. If John ate the apple, he'll be sick.
b. Antecedent: John ate the apple
c. Consequent: He'll be sick.

Claim A
In those circumstances where the first sentence (**the antecedent**) is true, the second sentence (**the consequent**) is true.

So the first two lines of the truth table make perfect sense. Claim A is safe when both sentences are true, and it is clearly false when the antecedent is true and the consequent is false.

But what about when the first sentence is false? Well, if he didnt eat the apple claim A is safe whether he's sick or not. Claim A only requires that IF he ate the apple sickness follows. So if he didnt, the claim is still compatible with the facts ("true"), according to the truth table.

Question: How well does this accord with our intutuions about conditional sentences in English (*if ... then ...)* ? Answer: Not very.

(4)

	Ant.	Cons.	Conditional	Truth table Truth Value
(a)	T	T	If 1960 was divisible by 5, then 1960 was a leap year.	T
(b)	F	T	If Al Gore won the election of 2000, George Bush won the election of 2004.	T
(c)	T	F	If George Bush won the election of 2004, Al Gore won the election of 2000.	F

(a) just seems false. (b) is weird not clear what kind of communicative act is being performed. (c) can be true as an instance of the "If X, I'll eat my hat" construction.

3 Statement Logic Classification of sentences

3.1 Tautologies

Sentences like $p \rightarrow (q \rightarrow p)$, parenthesized THIS way, are called tautologies, because they cant help but be true:

Rule

	p	q	$p \rightarrow q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	T
(d)	F	F	T

Example

	p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
(a)	T	T	T	T
(b)	T	F	T	T
(c)	F	T	F	T
(d)	F	F	T	T

4 Contingent sentences

Sentences like $(p \rightarrow q) \rightarrow p$, parenthesized THIS way, are called contingent sentences, They are sometimes true and sometimes false.

	p	q	$p \to q$	$(p \to q) \to p$
(a)	T	T	T	T
(b)	T	F	F	T
(c)	F	T	T	F
(d)	F	F	T	F

Another example:

	p	q	$p \to q$	$p \wedge (p \to q)$	$\neg q$	$(p \wedge (p \to q)) \to \neg q$
(a)	T	T	T	T	F	F
(b)	T	F	F	F	T	T
(c)	F	T	T	F	F	T
(d)	F	F	T	F	T	T

4.1 Contradictions

Some sentences have truth tables that always make them false. Such sentences are called contradictions, because they can't help but be false:

	p	$\neg p$	$p \wedge \neg p$
(a)	T	F	F
(b)	F	T	F

Another example:

	p	q	$p \to q$	$\neg(p \to q)$	$q \wedge \neg(p \to q)$
(a)	T	T	T	F	F
(b)	T	F	F	T	F
(c)	F	T	T	F	F
(d)	F	F	T	F	F

4.2 Logical Equivalence

Sometimes two distinct sentences have exactly the same truth values in all circumstances. Such sentences are **logically equivalent**.

(5) a. $p \rightarrow q$
b. $\neg p \vee q$
c.

	p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
(a)	T	T	F	T	T
(b)	T	F	F	F	F
(c)	F	T	T	T	T
(d)	F	F	T	T	T

d. $(\neg p \vee q) \Longleftrightarrow (p \rightarrow q)$