Logical Translations
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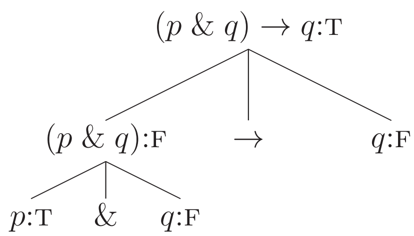
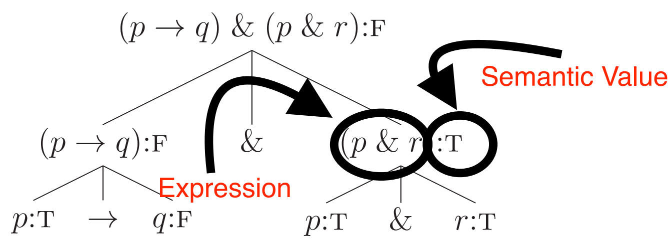
1 Problem 1: Trees

In this exercise, p is True, q is False, and r is True.

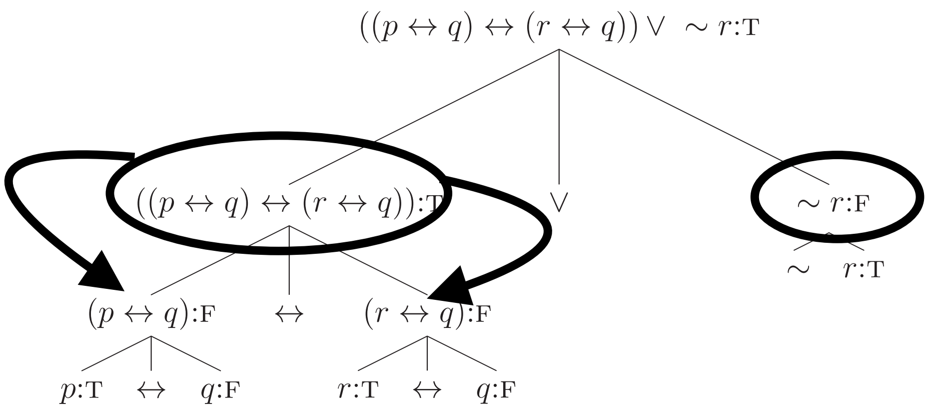
1a. $(p \mathbin{\&} q) \rightarrow q$



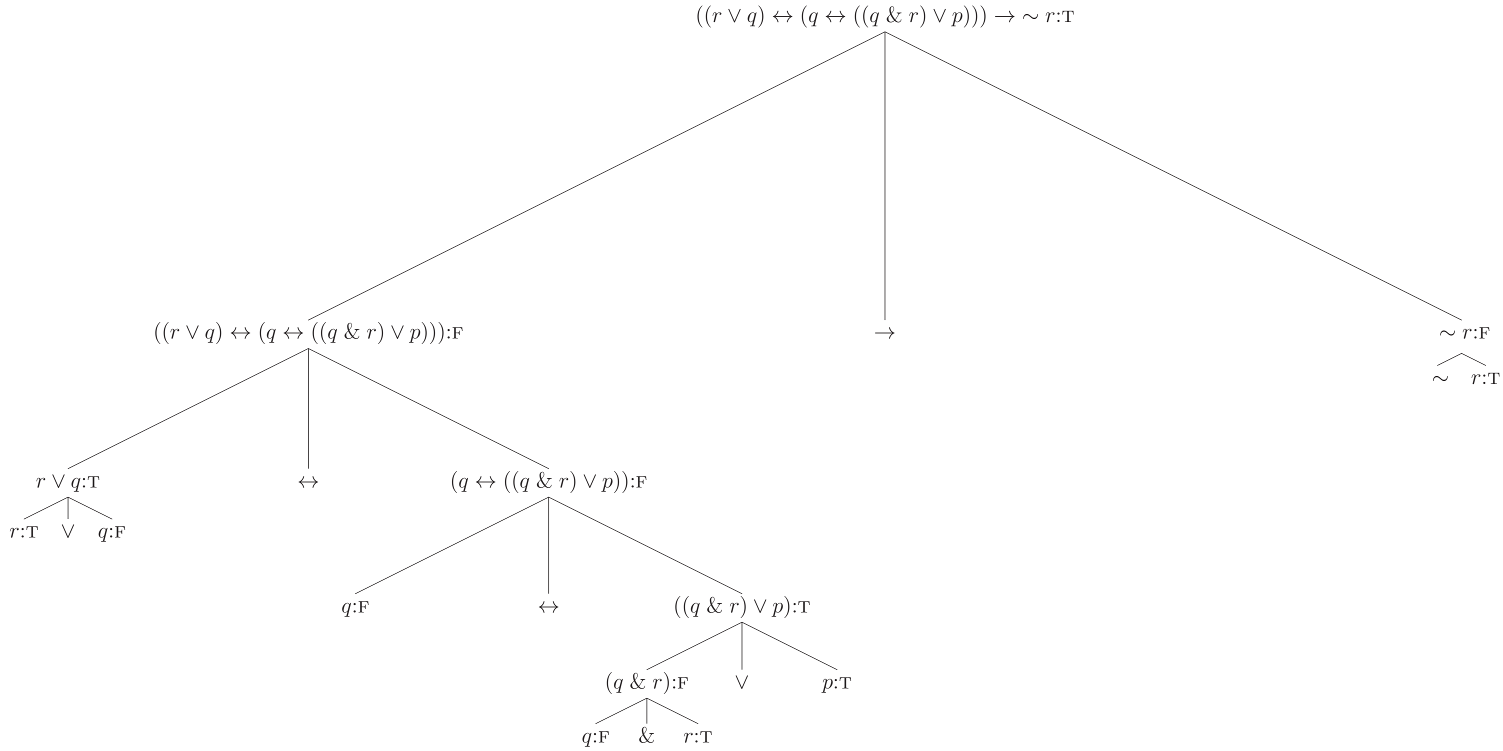
1d. $(p \rightarrow r) \mathbin{\&} (p \mathbin{\&} r)$



1f. $((p \leftrightarrow q) \leftrightarrow (r \leftrightarrow q)) \vee \sim r$



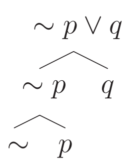
1h. $((r \vee q) \leftrightarrow (q \leftrightarrow ((q \,\&\, r) \vee p))) \rightarrow \sim r$



2 Problem 2: Truth tables

Common errors. Ignoring what parentheses (or the lack of them) tell you.

Note that the formula $p \vee q$ plays no role in the truth table for $\sim p \vee q$. This is because in the tree for $\sim p \vee q$, there is no node for $p \vee q$:



Notice that in the truth table for (a) there is a column for each node (unit, phrase) in the tree. $p \vee q$ is not a node in the tree, so there is no column for it. What the syntactic analysis (the tree) is telling you is that you do not have to compute the truth value of $p \vee q$ in order to know the truth value of $\sim p \vee q$.

In contrast look what happens to the tree when we add parentheses:

- $\sim(p \vee q)$
  - $\sim$
  - $(p \vee q)$
    - p
    - $\vee$
    - q

In this case you do have to compute the truth value of $p \vee q$. the truth table would be

p	q	$(p \vee q)$	$\sim(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Another serious error is leaving out p and q as the first columns, and starting with columns for $\sim p$ or $\sim q$ or both, as in the following.

$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	T
T	F	T
F	T	F
F	F	T

What was done here is in some sense correct. The problem is that actually the order of the rows in the truth table has been scrambled, and you don't know it. If we put back the p and q columns, we can see that in fact the rows are in reverse order from the usual order:

p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	F
T	T	F	F	T

When the row order gets scrambled like this, it's very hard (or impossible) to see that actually $\sim q \rightarrow \sim q$ is equivalent to $\sim p \vee q$ (shown in the correctly done truth tables below).

Now the answers to the given exercises.

a.

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b.

p	q	$\sim q$	$p \vee \sim q$	$\sim (p \vee \sim q)$
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

c.

p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

The big news is that the truth tables for $\sim p \vee q$ and $\sim q \rightarrow \sim p$ are the same! Therefore these two logical expressions are logically equivalent.

3 Problem 9

9*a*.	John gave \$10 to Mary.	GIVE(j, **t**, **m**)
9*b*.	Mary was given \$10 by John.	GIVE(**j**, **t**, **m**)
9*c*.	Toby was under the table	UNDER(**t**, **b**)
	b is the table	
9*d*.	Clive showed Maddy the photoes	SHOW(**c**, **p**, **m**)
	p is the photos.	
9*e*.	China is east of Europe	EAST-OF(**c**, **e**)
	EAST(**c**, **e**)	
	equally good answers	
9*f*.	Sheila is a surgeon	SURGEON(**s**)
9*g*.	PAINT(**b**, **k**)	
	k is the kitchen	
9*h*.	Bill was painting in the kitchen.	PAINT2(**b**)
9*i*.	Mary finally bought the painting yesterday.	BUY(**m**, **p**)
	p is the painting	

Notice: All these answers are based on understanding the morphology and syntactic structure. You need to know that *gave* is the past tense of *give* to know that the predicate in (a) is GIVE. You need to know that *given* is the passive of *give* in order to know that the predicate in (b) is also GIVE, and that what shows up in subject position is the recipient of the giving and that what shows up in the *by*-phrase is the agent of the giving, which tells you what argument positions they take with GIVE. If you think *$10 to Mary* is an NP, then you think that GIVE is a 2-argument predicate which takes *, John* and *$10 to Mary* as its arguments, and you haven't got a prayer of discovering the truth, which is that GIVE is a 3-argument predicate that might as well be named GIVE-TO, and that its arguments in (a) are *John*, *$10*, and *Mary*.

You also need to understand the syntactic structure inorder to know that *finally* in (i) is an adjunct; it looks a little like a temporal adjunct but it's actually a more complicated adverbial meaning to be discussed at another time.

4 Problem 11

Comments on this problem. Two kinds of issues you should definitely address:

4.0.1 The verb *be* is not a predicate. No form of the verb *be* is a predicate. Given *Frank is generous* as an English sentence, the following is a XXX-bad translation:

$$\textsc{be}(\mathrm{f}, \mathrm{g})$$

The correct translation involves identifying the word *generous* as an adjective and knowing that adjectives are predicates.

$$\textsc{generous}(\mathrm{f})$$

4.0.2 I got a lot of answers that looked like this:

Frank is not both rich and generous
$\textsc{rich} \vee \textsc{generous}(\sim \mathrm{f})$

Neither Bill nor Alice laughed.]
$\sim \textsc{laugh}(\mathrm{b} \vee \mathrm{a})$

Either Sydney or Canberra is the capital of Australia.]
$\sim$ CAPITAL-OF(s $\vee$ c, a)

All of these are triple X wrong. You've forgotten what we did when we did statement logic. We paraphrased these kinds of sentences into sentences in which "and", "or", "not" applied to sentences (see below). Then you translated the sentences into statement logic using these paraphrases. The reason you did that is that the logical connectives &, $\vee$ and $\sim$ can **only** apply to statements. This is still true even though we are now doing predicate logic. All of the above answers are triple XXX wrong because they apply &, $\vee$ or $\sim$ to something that is not a statement. Predicates are not statements so

RICH $\vee$ GENEROUS

does not make sense.

Constants that translate proper names (like f, a, b) are not statements, so

GENEROUS($\sim$ f)

does not make sense. In fact it's a syntax error in the language of logic. Predicate expressions take individual-denoting expressions as their arguments, because predicates are true or false of individuals. What individual is denoted by $\sim$ f?. The individual you get by mushing together everything in the universe but Frank? But that individual, even if it made sense, simply does not arise in the semantics of ordinary English. What does arise is

$\sim$ GENEROUS(f)

Which is a statement that is true whenever generous(f) is false. In other words, &, $\vee$ and $\sim$ work **exactly** as they did in statement logic because predicate logic expressions like

GENEROUS(f)

are just statements. They denote true or false just like our ps and qs did in statement logic. That's why we learned statement logic, to prepare us for the frostier regions of predicate logic: &, $\vee$ and $\sim$ are the connectives that hold together the elements of our logical forms. Keep using them the same way you did before (to connect statements).

The expression

$$\textsc{capital-of}(\mathrm{s} \vee \mathrm{c},\ \mathrm{a})$$

is nonsense for the same reason generous($\sim$ f) is. The connective $\vee$ connects only sentences, not individuals, The predicate capital-of is true only of pairs of individuals where the first is the capital of the second. But what individual is denoted by s $\vee$ c? I don't know. Let's stick with the perfectly good interpretation we have for $\vee$, that it connects statements.

These are the right answers (with helpful paraphrases):

11*a*. Either Sydney or Canberra is the capital of Australia.
Either Sydney is the capital of Australia or
Canberra is the capital of Australia.
$\textsc{capital-of}(\mathbf{s}, \mathbf{a}) \vee \text{capital-of}(\mathbf{c}, \mathbf{a})$

11*b*. Alice didn't laugh and Bill didn't either.
Alice didn't laugh and Bill didn't laugh either.
$\sim \textsc{laugh}(\mathbf{a})\ \&\ \ \sim \textsc{laugh}(\mathbf{b})$
$\sim (\textsc{laugh}(\mathbf{a}) \vee \textsc{laugh}(\mathbf{b}))$
Equivalent answers

11*c*. Frank is not both rich and generous
$\sim (\textsc{rich}(\mathbf{f})\ \&\ \textsc{generous}(\mathbf{f}))$

11*d*. Gina will marry Leo or Fred
Gina will marry Leo or Gina will marry Fred
$\textsc{marry}(\mathbf{g}, \mathbf{l}) \vee \textsc{marry}(\mathbf{g}, \mathbf{f})$

11*e*. Alice didn't laugh and nor did Bill.
$\sim \textsc{laugh}(\mathbf{a})\ \&\ \ \sim \textsc{laugh}(\mathbf{b})$
$\sim (\text{laugh}(\mathbf{a}) \vee \text{laugh}(\mathbf{b}))$
Equivalent answers

11*f*. Skipped!

11*g*. Neither Bill nor Alice laughed.
$\sim (\textsc{laugh}(\mathbf{b}) \vee \text{laugh}(\mathbf{n}))$
$\sim \textsc{laugh}(\mathbf{b})\ \&\ \sim \text{laugh}(\mathbf{n})$
Equivalent answers

12*h* Skipped!

5 Problem 12

a. go (a, m) & [visit (a, r) $\vee$ interview (a, c)]
b. [go (a, m) & visit (a, r)] $\vee$ interview (a, c)

The following partial truth table shows that (a) and (b) are not equivalent.

p	q	r	q $\vee$ r	a. p & (q $\vee$ r)	p & q	b. (p & q) $\vee$ r
f	t	t	t	f	f	t

6 Problem 15

In this problem, a number of people misunderstood the directions and simplified classified all the adverbials as either time, place, or manner. It's nice to know you have this skill but the point of this exercise lay elsewhere. What your previous grammar classes may not have revealed to you is that phrases expressing Time, Place, and Manner can sometimes be **arguments of verbs** (which means they belong in the logical form as arguments or the predicate). Your job in this exercise was to find the time, place, and mannner phrases that were arguments.

Two examples are *in marble halls* in (a); and *that night* in (e). There are two tests that compel us to call something an argument. First, if something is an **obligatory** modifier of a verb, we call it an argument. The sentence *They dwelt* sounds wrong or incomplete, so we say the location is an obligatory argument of the verb *dwell*. This is consistent with intuition that the meaning of the verb highlights or profiles the location (the verb *reside* is similar). Or to put it another way, the verb denotes a relation between an individual and the place they live. The second principle is that a direct object is always an argument of the verb. Now a good first approximation of what a direct object is in English is that it is the Noun Phrase immediately following the verb. But that's not always the case. There are **adverbial NPs** modifying verbs too. In *John sings every Wednesday*, *every Wednesday* is the first NP following the verb but it is not the direct object. It is an adverbial (which means it is an adjunct) expressing Time, that is, telling us when John sings. We can see that it's not a direct object because it can't passivize and keep the same meaning:

? Every Wednesday was sung by John.

Now consider:

f. John carefully planned that night.
1. John carefully planned the details of everything that would take place that night.
2. John carefully planned something. The planning took place that night.
f' That night was carefully planned by John.

Sentence (f) is ambiguous between a reading (reading 1) on which *that night* is a direct object (and therefore an argument), and a reading (reading 2) on which it is an adjunct (telling us when the planning occurred). The passive (f') can only mean the details of the night were carefully planned. That is, it is a paraphrase of reading (1), supporting that the idea that in reading (1), *that night* is a direct object, and in reading (2) it is not; that means that in reading (1) *that night* is an **argument**, and in reading (2) it is not.

For clarity, **all** arguments have been italicized, including manner, place, and time arguments. Notice subjects and objects are **always** arguments.

a. *They* dwelt *in marble halls.*
b. *The theremin* echoes marvelously in marble halls.
c. *John* behaved *impeccably.* [*impeccably* has been called an argument because the verb *behave* always requires the understanding or the expression of a specific manner of behavior. Although we can say *John behaved*, so that the manner is not syntactically obligatory, we understand that sentence as meaning *John behaved well (or at least acceptably)*; that is, we supply a specific manner of behavior as the default. So since a specific manner is always understood with any use of *behave*, we say the manner is an argument of the verb. Notice the difference with *John danced.* Although I suppose it's true that there had to be **some** manner in which John danced (*well* or *badly*, *wildly* or *with great control*), we don't come away after hearing the sentence with any idea of what that manner was. So it is not the case that a specific manner of dancing is supplied with any use of the verb *dance*. Conclusion: In *John* behaved *impeccably impeccably* is an argument; in *, John danced wildly*, *wildly* is an adjunct.
d. *John* carelessly lost *the car keys.*
e. *Simon* carefully planned *the weekend* that night. The NP *the weekend* is the direct object (what's planned), certainly an argument!)

f. 1. *Simon* carefully planned that night.
   2. *Simon* carefully planned *that night*.
   This sentence is is ambiguous. On reading 2 *that night* is the direct object, certainly an argument!. Reading 2 is appropriate in a context like the following: "Simon knew his wedding night was going to be the most important night of his life. Being a methodical man, he spent two weeks doing what he always did with important events. Simon carefully planned that night."

g. *The meetings* lasted *all day*. [This one is the most debatable, and interesting, syntactically. The phrase *all day* looks like an NP but doesn't passivize. Yet the meaning of the verb *last* appears to involve measuring the duration of an event. Leaving off the *all day* in this sentence changes the meaning to something strange. This case seems to resemble the case of *behave* in which an adverbial specification is semantically obligatory; hence *all day* is an argument.]

h. *The elephants* were upset and nervous all day.