

Property Definitions: Russell

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Sets defined by properties: I

Set of elephants

$$\text{ELEPHANT} = \{x \mid \text{elephant}(x)\}$$

ELEPHANT contains no sets. In particular, it does not contain itself!

$$\text{ELEPHANT} \notin \text{ELEPHANT}$$

Sets defined by properties: II

Set of abstract concepts

$$AC = \{x \mid \text{abstract-concept}(x)\}$$

AC contains itself!

$$AC \in AC$$

The Russell property

$$U = \{x \mid x \notin x\}$$

The paradox

$$U = \{x \mid x \notin x\}$$

1. Either $U \in U$ or $U \notin U$. (Definition of set)
2. If $U \in U$ then $U \notin U$. (Definition of U). **Contradiction!**
3. If $U \notin U$ then $U \in U$. (Definition of U). **Contradiction!**

Grelling's Paradox

AUTOLOGICAL = $\{x \mid x \text{ is an adjective that describes itself}\}$

HETEROLOGICAL = $\{x \mid x \text{ is an adjective that does not describe itself}\}$

1. The adjective *short* is autological.
2. The adjective *long* is heterological.
3. What about the adjective *heterological*?
4. What about the adjective *autological*?