Rules, Functions, and Recursive Definitions

1 Rules as functions

Consider the English past tense, phonetically. A **citation form** is the form we find in the dictionary. The citation form of *walk* is /wɔk/. The past tense form is /wɔkt/ (spelled *walked*). So you add /t/:

> If α is a verb citation form, then α+/t/ is the past tense of the verb.

There are three problems with this. It doesn't work for verbs that end with voiced sounds; the past tense of *hug* is not /hʌgt/; it's /hʌgd/. It doesn't work for verbs that end with /d/ or /t/. The past tense of *raid* is /redəd/ and the past tense of *sight* is /saitəd/. Finally, it doesn't work for irregular verbs. The past tense of *sing* is not /sɪŋt/. It's /saeŋ/ (spelled *sang*).

Let's leave the irregulars out of it and fix the regulars:

1. If α is a regular verb citation form, and α ends in /t/ or /d/, then α+/əd/ is the past tense of the verb; otherwise,

2. if α is a verb citation form, and α ends in a voiceless sound, then α+/t/ is the past tense of the verb;

3. otherwise, if α is a verb citation form, then α+/d/ is the past tense of the verb.

This defines a **function**. Let's call the function **Past** and let's call the set of regular verb citation forms $\mathbf{Verb}_{\text{reg}}$. Here's how the function definition looks in our textbook's notation:

$$\text{Past} = \{\langle \alpha, \alpha + \text{suf}\rangle \mid \begin{array}{l} \alpha \in \text{Verb}_{\text{reg}} \text{ and} \\ \text{suf} = /\text{əd}/ \text{ if } \textsc{end}(\alpha) \in \{/\text{t}/, /\text{d}/\}; \text{ and} \\ \text{suf} = /\text{t}/ \text{ if } \textsc{end}(\alpha) \in \text{Voiceless; and} \\ \text{suf} = /\text{d}/ \text{ otherwise}\} \end{array}$$

I'm assuming that END is itself a function that for each verb stem, returns the last sound in it. So, for example:

$$\textsc{end}(/\text{wɔk}/) = /\text{k}/$$

Given this definition of **Past**, it's now legitimate to write:

Past(/red/) = redəd
Past(/wɔk/) = wɔkt
Past(/hʌg/) = hʌgd

2 Reviewing Recursive Definitions

Defining the Natural numbers:

(i) $0 \in \aleph$
(ii) If $x \in \aleph$ then $\text{successor}(x) \in \aleph$
(iii) Nothing else is in $\aleph$.

Is 3 a natural number....?

1. 0 is a natural number. (Axiom i).
2. successor(0)=1. (Def of successor function)
3. 1 is a natural number (Axiom ii on steps 1 and 2)
4. successor(1)=2. (Def of successor function)
5. 2 is a natural number (Axiom ii on steps 3 and 4)
6. successor(2)=3. (Def of successor function)
7. 3 is a natural number (Axiom ii on steps 5 and 6) Q. E. D.

This is called a recursive definition. In order to define what's in the set $\aleph$ I make reference to what's in the set $\aleph$ (clause ii).

The official Definition of $\aleph$ (Peano's definition):

(i) $0 \in \aleph$
(ii) If $x \in \aleph$ then $\text{successor}(x) \in \aleph$
(iii) $\aleph$ is the smallest set that satisfies clause (i) and (ii)

Sufficient and necessary conditions required. Clause (i) and (ii) alone aren't enough to keep Bill Clinton out of the set of natural numbers.

3 Recursive definitions of grammars

Definition of a small language recursively by defining the set of sentences of the language, which we call S.

I'll use 'xy' to mean 'x' followed by (or concatenated with) 'y':

1. First we define a set N as follows:
2. Let $\mathrm{N} = \{$book, magazine, boy, girl$\}$
3. Let $\mathrm{Adj} = \{$big, fat$\}$
4. Let $\mathrm{V} = \{$liked, loved$\}$
5. Let $\mathrm{Art} = \{$the$\}$.
6. Next we define a set Nom:
   (a) If $x \in \mathrm{N}$ then $x \in \mathrm{Nom}$.
   (b) If $x \in \mathrm{Adj}$ and $y \in \mathrm{Nom}$ then $xy \in \mathrm{Nom}$.
   (c) Nothing else is in Nom.
7. Next we define a set NP, using Nom:
   (a) If $x \in \mathrm{Art}$ and $y \in \mathrm{Nom}$ then $xy \in \mathrm{NP}$.
   (b) Nothing else is in NP.
8. Next we define a set VP:
   (a) If $x \in \mathrm{V}$ and $y \in \mathrm{NP}$ then $xy \in \mathrm{VP}$.
   (b) Nothing else is in VP.
9. Finally we define the set S, the set of sentences of the language.
   (a) If $x \in \mathrm{NP}$ and $y \in \mathrm{VP}$ then $xy \in \mathrm{S}$.
   (b) If $x \in \mathrm{NP}$ and $y \in \mathrm{S}$ then $'x$ believed $y' \in \mathrm{S}$.
   (c) Nothing else is in S.

Questions

1. How would a linguist define this language using phrase structure rules?

2. Is S infinite?

3. Which of the following are in Nom?

   boy
   big boy
   big fat boy
   fat big boy
   the big fat boy

4. Is the following in S?

   The big boy believed the girl believed the boy liked the big fat book.