Predicate Logic

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Predicate Logic

The notion Predicate

- A finer grained analysis of statements than statement logic.
- We divide up assertions into predicates and arguments:

	walk(1)	see(2)	give(3)
john	walk(j)	see(j,m)	give(j,m,b)
mary	walk(m)	see(m,m)	give(m,m,b)
bean37	walk(b37)	see(b37,m)	give(b37,m,b)

- Predicates for nouns as well as verb

	man	woman	bean
john	man(j)	woman(j)	bean(j)
mary	man(m)	woman(m)	bean(m)
bean37	man(b37)	woman(b37)	bean(b37)

Statements, Individuals, Properties

- Statements *predicate* properties of individuals.

John	walks.
Individual	Property

- The term property corresponds roughly to our notion of a 1-place relation but is more general because properties can be complex.

John	loves Mary.
Individual	Property
Subject	Predicate

- Some statements don't seem to involve any individuals!

Every man walks.

Quantified statements

- Quantifiers ($\forall$, $\exists$)

All beans walk.

$\forall x[\text{bean}(x) \rightarrow \text{walk}(x)]$

A bean walks!

$\exists x[\text{bean}(x) \wedge \text{walk}(x)]$

- Syntactically these are determiners!

$\forall$	every, all, each, any
$\exists$	a(n), some, any(!)
?	no
?	the
?	3
?	most

Translating Every man walks

$$\forall x[\text{man}(x) \rightarrow \text{walk}(x)]$$

- Why *man*? Translates the noun “man”.

Translating Every man walks

$$\forall x[\text{man}(x) \rightarrow \text{walk}(x)]$$

- Why *man*? Translates the noun “man”.
- Why *walk*? Translates the verb “walks”.

Translating Every man walks

$$\forall x[\text{man}(x) \rightarrow \text{walk}(x)]$$

- Why *man*? Translates the noun “man”.
- Why *walk*? Translates the verb “walks”.
- Why $\forall$? Translates the determiner “every”.

Translating Every man walks

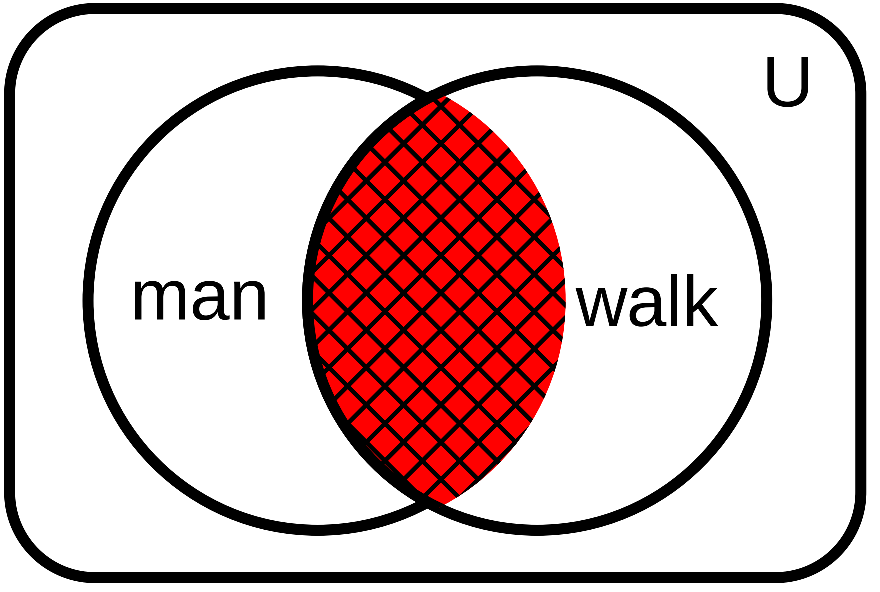
$$\forall x[\text{man}(x) \rightarrow \text{walk}(x)]$$

- Why *man*? Translates the noun “man”.
- Why *walk*? Translates the verb “walks”.
- Why $\forall$? Translates the determiner “every”.
- Why $\rightarrow$? ?????

Translating Every man walks

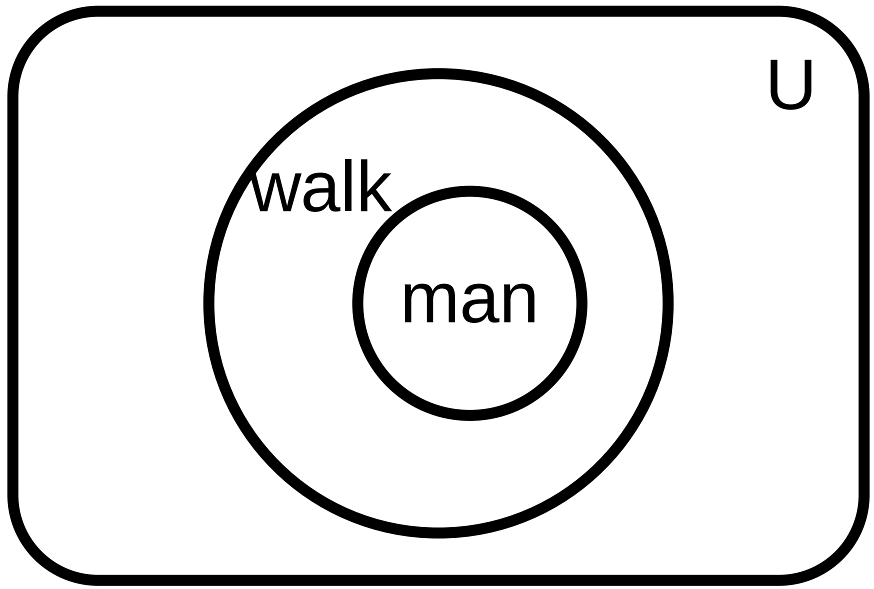
$$\forall x[\text{man}(x) \rightarrow \text{walk}(x)]$$

- Why *man*? Translates the noun “man”.
- Why *walk*? Translates the verb “walks”.
- Why $\forall$? Translates the determiner “every”.
- Why $\rightarrow$? ?????
- We got something wrong!



$[\![\text{man}]\!] \cap [\![\text{walk}]\!] \neq \emptyset$

Some man walks



$[\![\text{man}]\!] \cap [\![\text{walk}]\!] = [\![\text{man}]\!]$

Every man walks

Second Try

- *Every* needs to combine with 2 properties to make a statement.

Every P Q's

- So, really our proposed translation of *every* is not $\forall$ but

$$\forall x[\underline{\qquad}(x) \rightarrow \underline{\qquad}(x)]$$

- But why is $\rightarrow$ right? Why not $\wedge$? $\vee$?

Why is material implication part of the meaning of every?

- Every man walks

$$[\![\text{man}]\!] \subseteq [\![\text{walk}]\!]$$

  Every member of the man set is a member of the walker set.

- Statements with $\forall x$ are statements about every individual in the universe.

  Wrong! $\forall x[\text{man}(x) \wedge \text{walk}(x)$

  Everything is a man and walks

- Cases

x not a man	man(x)	$\rightarrow$	walk(x)	T
	FF			
	F		T	
xa walking man	man(x)	$\rightarrow$	walk(x)	T
	TT			
xa nonwalking man	man(x)	$\rightarrow$	walk(x)	F
	TF			

Aint logic funny

- The *contrapositive* is logically equivalent

$$(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$$

- The corresponding universally quantified sentences are logically equivalent:

$$\forall x[\mathsf{P}(x) \rightarrow \mathsf{Q}(x)] \Leftrightarrow \forall x[\neg \mathsf{Q}(x) \rightarrow \neg \mathsf{P}(x)]$$

- Logically equivalent

Every man walks $\Leftrightarrow$ Every non-walker is a non man.
Every raven is black $\Leftrightarrow$
Every non-black thing is a non raven.

Statement Logic Laws I

1. Idempotent Laws

(a) $(P \vee P) \Longleftrightarrow P$

(b) $(P \wedge P) \Longleftrightarrow P$

2. Associative Laws

(a) $((P \vee Q) \vee R) \Longleftrightarrow$

$(P \vee (Q \vee R))$

(b) $((P \wedge Q) \wedge R) \Longleftrightarrow$

$(P \wedge (Q \wedge R))$

3. Commutative Laws

(a) $(P \vee Q) \Longleftrightarrow (Q \vee P)$

(b) $(P \wedge Q) \Longleftrightarrow (Q \wedge P)$

4. Distributive Laws

(a) $((P \vee Q) \wedge (P \vee R)) \Longleftrightarrow$

$(P \vee (Q \wedge R))$

(b) $((P \wedge Q) \vee (P \wedge R)) \Longleftrightarrow$

$(P \wedge (Q \vee R))$

5. Identity Laws

(a) $(P \vee F) \Longleftrightarrow P$

(b) $(P \vee T) \Longleftrightarrow T$

(c) $(P \wedge F) \Longleftrightarrow F$

(d) $(P \wedge T) \Longleftrightarrow P$

Statement Logic Laws II

6. ComplementLaws

(a) $(P \vee \neg P) \Longleftrightarrow T$

(b) $\neg\neg P \Longleftrightarrow P$

(c) $(P \wedge \neg P) \Longleftrightarrow F$

7. De Morgan's Laws

(a) $\neg(P \vee Q) \Longleftrightarrow (\neg Q \wedge \neg P)$

(b) $\neg(P \wedge Q) \Longleftrightarrow (\neg Q \vee \neg P)$

8. Conditional Laws

(a) $(P \rightarrow Q) \Longleftrightarrow (\neg P \vee Q)$

(b) $(P \rightarrow Q) \Longleftrightarrow (\neg Q \rightarrow \neg P)$

(c) $(P \rightarrow Q) \Longleftrightarrow \neg(Q \wedge \neg P)$

9. Biconditional Laws

(a) $(P \leftrightarrow Q) \Longleftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P))$

(c) $(P \leftrightarrow Q) \Longleftrightarrow ((\neg Q \wedge \neg P) \wedge (P \wedge Q))$

Predicate Logic Laws: Quantifier negation

Law 1	$\sim (\forall x)\phi(x) \Leftrightarrow (\exists x) \sim \phi(x)$
Law 1'	$(\forall x)\phi(x) \Leftrightarrow \sim (\exists x) \sim \phi(x)$
Law 1"	$\sim (\forall x) \sim \phi(x) \Leftrightarrow (\exists x)\phi(x)$
Law 1"'	$(\forall x) \sim \phi(x) \Leftrightarrow \sim (\exists x)\phi(x)$

Deriving (1') from (1)

Law 1	$\sim (\forall x)\phi(x) \Leftrightarrow (\exists x) \sim \phi(x)$
	$\sim\sim (\forall x)\phi(x) \Leftrightarrow \sim (\exists x) \sim \phi(x)$
Law 1'	$(\forall x)\phi(x) \Leftrightarrow \sim (\exists x) \sim \phi(x)$

Quantifier negation and De Morgan's Law

De Morgan's Law

$$p \wedge q \Leftrightarrow \sim [\sim p \vee \sim q]$$

Universal Quantification

$$\begin{aligned} \forall x \phi(x) \quad \leftrightarrow \quad & \phi(a) \wedge \phi(b) \wedge \phi(c) \cdots \wedge \phi(z) \\ & \sim [\sim \phi(a) \vee \sim \phi(b) \vee \sim \phi(c) \cdots \vee \sim \phi(z)] \\ & \sim \exists x \sim \phi(x) \end{aligned}$$

Predicate Logic Laws: Quantifier Distribution

Law 2 $(\forall x)\phi(x) \wedge \psi(x) \Leftrightarrow (\forall x)\phi(x) \wedge (\forall x)\psi(x)$

Law 3 $(\exists x)\phi(x) \vee \psi(x) \Leftrightarrow (\exists x)\phi(x) \vee (\exists x)\psi(x)$

Law 4 $(\forall x)\phi(x) \vee (\forall x)\psi(x) \Rightarrow (\forall x)(\phi(x) \vee \psi(x))$

Law 5 $(\exists x)(\phi(x) \wedge \psi(x)) \Rightarrow$

$$(\exists x)\phi(x) \wedge (\exists x)\psi(x)$$

Laws of Quantifier Movement

Law 9 $\phi \rightarrow (\forall x)\psi(x) \Leftrightarrow (\forall x)\phi \rightarrow \psi(x)$

provided that x is not free in ϕ

Law 10 $\phi \rightarrow (\exists x)\psi(x) \Leftrightarrow (\exists x)\phi \rightarrow \psi(x)$

provided that x is not free in ϕ

Law 11 $\forall\phi(x) \rightarrow \psi \Leftrightarrow (\exists x)(\phi(x) \rightarrow \psi$

provided that x is not free in ψ

Law 12 $\exists\phi(x) \rightarrow \psi \Leftrightarrow (\forall x)(\phi(x) \rightarrow \psi$

provided that x is not free in ψ

Prenex Normal Form

(1)		$\boxed{(\exists x)\mathsf{F}(x)} \rightarrow \boxed{(\forall y)\mathsf{G}(y)}$
(2)	Law 9 on (1)	$(\forall y)[(\exists x)\mathsf{F}(x) \rightarrow \mathsf{G}(y)]$
(3)	Law 12 on (2)	$(\forall y)[(\forall x)(\mathsf{F}(x) \rightarrow \mathsf{G}(y))]$
(4)	Law 12 on (1)	$(\forall x)[\mathsf{F}(x) \rightarrow (\forall y)\mathsf{G}(y)]$
(5)	Law 9 on (4)	$(\forall x)(\forall y)[\mathsf{F}(x) \rightarrow \mathsf{G}(y)]$
	Law 6	(3) and (5) equivalent

Universal Instantiation

$$\frac{\forall x \phi(x)}{\phi(v)}$$

$$\begin{array}{ll} (a) & \text{All men are mortal} \\ (b) & \text{Socrates is a man} \\ \hline (c) & \text{Socrates is mortal} \end{array}$$

(1)	$(\forall x)(\mathsf{H}(x) \rightarrow \mathsf{M}(x))$	(a)
(2)	$\mathsf{H}(s)$	(b)
(3)	$(\mathsf{H}(s) \rightarrow \mathsf{M}(3))$	(1) U.I.
(4)	$\mathsf{M}(s)$	$(2), (3)$ M.P.$(= c)$

Universal generalization

Instantiation

$$\frac{\forall x \phi(x)}{\phi(v)}$$

Act
on
$\Leftrightarrow$ same
variables

Generalization

$$\frac{\phi(v)}{\forall x \phi(x)}$$

Existential generalization and instantiation

Instantiation	Act on ⇔ same variables	Generalization
$\dfrac{\exists x \phi(x)}{\phi(c)}$		$\dfrac{\phi(c)}{\exists x \phi(x)}$

Restriction: Constant introduced by E.I. cannot have occurred earlier in the same proof.

Invalid use of E.G.

(a)	Some philosophers are dogged.
(b)	Some dogs are rabid.
(c)	Some philosophers are rabid

(1)	$(\exists x)(\mathsf{P}(x) \wedge \mathsf{G}(x))$	
(2)	$(\exists y)(\mathsf{D}(y) \wedge \mathsf{R}(y))$	
(3)	$(\mathsf{P}(c) \wedge \mathsf{G}(c))$	E. G. on (1)
(4)	$(\mathsf{D}(c) \wedge \mathsf{R}(c))$	E. G. on (2)
(5)	$\mathsf{P}(c)$	Simplification on (3)
(6)	$\mathsf{R}(c)$	Simplification on (4)
(7)	$\mathsf{P}(c) \wedge \mathsf{R}(c)$	Conjunction on $(5), (6)$
(8)	$\exists x(\mathsf{P}(x) \wedge \mathsf{R}(x))$	E. G. on $(7)[= (c)]$

Invalid use of E.G.

(a)	Some philosophers are dogged.
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(c)	Some philosophers are rabid

(1)	$(\exists x)(\mathsf{P}(x) \wedge \mathsf{G}(x))$		
(2)	$(\exists y)(\mathsf{D}(y) \wedge \mathsf{R}(y))$		
(3)	$(\mathsf{P}(c) \wedge \mathsf{G}(c))$	E. G. on (1)	
(4)	$(\mathsf{D}(c) \wedge \mathsf{R}(c))$	E. G. on (2)	This step is bad!
(5)	$\mathsf{P}(c)$	Simplification on (3)	
(6)	$\mathsf{R}(c)$	Simplification on (4)	
(7)	$\mathsf{P}(c) \wedge \mathsf{R}(c)$	Conjunction on $(5), (6)$	
(8)	$\exists x(\mathsf{P}(x) \wedge \mathsf{R}(x))$	E. G. on $(7)[= (c)]$	

Proof 1

(a)	$\sim (\exists x)(\mathsf{P}(x) \wedge \mathsf{Q}(x))$	
(b)	$(\exists x)(\mathsf{P}(x) \wedge \mathsf{R}(x))$	
(c)	$\exists x(\mathsf{R}(x) \wedge \sim \mathsf{Q}(x))$	

(1)	$(\forall x) \sim (\mathsf{P}(x) \wedge \mathsf{Q}(x))$	Law 1 on (a)
(2)	$(\mathsf{P}(c) \wedge \mathsf{R}(c))$	E.I. on (b)
(3)	$\sim (\mathsf{P}(c) \wedge \mathsf{Q}(c))$	U.I. on (1)
(4)	$(\mathsf{P}(c) \rightarrow \sim \mathsf{Q}(c))$	Cond. on (3)
(5)	$\mathsf{P}(c)$	Simplification on (2)
(6)	$\sim \mathsf{Q}(c)$	M.P. on $(4), (5)$
(5)	$\mathsf{R}(c)$	Simplification on (1)
(6)	$(\mathsf{R}(c) \wedge \sim \mathsf{Q}(c))$	Conj. on $(4), (5)$
(7)	$(\exists x)(\mathsf{R}(x) \wedge \sim \mathsf{Q}(x))$	E. G. on (6)

Proof 2

(a)	$(\forall x)(\mathsf{P}(x) \to \mathsf{Q}(x))$	
(b)	$(\exists x)(\mathsf{P}(x) \wedge \mathsf{R}(x))$	
(c)	$\exists x(\mathsf{R}(x) \wedge \mathsf{Q}(x))$	

(1)	$(\mathsf{P}(c) \wedge \mathsf{R}(c))$	E.I. on (b)
(2)	$(\mathsf{P}(c) \to \mathsf{Q}(c))$	U.I. on (a)
(3)	$\mathsf{P}(c)$	Simplification on (1)
(4)	$\mathsf{Q}(c)$	M.P. on $(3), (2)$
(5)	$\mathsf{R}(c)$	Simplification on (1)
(6)	$(\mathsf{R}(c) \wedge \mathsf{Q}(c))$	Conj. on $(4), (5)$
(7)	$(\exists x)(\mathsf{R}(x) \wedge \mathsf{Q}(x))$	E. G. on (6)

Proof 3

(a)	$(\forall x)\mathsf{P}(x) \rightarrow \mathsf{Q}(x)$	
(b)	$\sim (\forall x)\mathsf{P}(x) \rightarrow \mathsf{R}(x)$	
(c)	$\exists x(\sim \mathsf{R}(x) \wedge \mathsf{Q}(x))$	

(1)	$(\exists x) \sim \mathsf{P}(x) \rightarrow \mathsf{R}(x)$	Law (1) on(b)
(2)	$\sim (\mathsf{P}(c) \rightarrow \mathsf{R}(c))$	E.I. on (1)
(3)	$\sim\sim (\mathsf{P}(c) \wedge \sim \mathsf{R}(c))$	Cond. on (2)
(4)	$(\mathsf{P}(c) \wedge \sim \mathsf{R}(c))$	Negation on (3)
(5)	$\sim \mathsf{R}(c)$	Simplification on (4)
(6)	$\mathsf{P}(c)$	Simplification on (4)
(7)	$\mathsf{P}(c) \rightarrow \mathsf{Q}(c)$	U.I. on (a)
(8)	$\mathsf{Q}(c)$	M.P. on (7)
(9)	$\sim \mathsf{R}(c) \wedge \mathsf{Q}(c)$	Conj. on $(5), (8)$
(10)	$(\exists x)(\sim \mathsf{R}(x) \wedge \mathsf{Q}(x))$	E. G. on (9)

Babies are illogical

(a) Babies are illogical.

$$\forall x(\mathsf{B}(x) \rightarrow \mathsf{I}(x))$$

(b) Nobody who is despised can manage a crocodile.

$$\sim \exists x[\mathsf{D}(x) \wedge \exists y(\mathsf{C}(y) \wedge \mathsf{M}(x,y))]$$

(c) Illogical persons are despised.

$$\forall x(\mathsf{I}(x) \rightarrow \mathsf{D}(x))$$

(d) Babies cannot manage crocodiles.

$$\forall x(\mathsf{B}(x) \rightarrow \forall y(\mathsf{C}(y) \rightarrow \sim \mathsf{M}(x,y)))$$

Inability to manage crocodiles

(1)	$(\mathsf{B}(v_1) \to \mathsf{I}(v_1))$	U.I. on (a)
(2)	$\forall x \sim [\mathsf{D}(x) \wedge \exists y(\mathsf{C}(y) \wedge \mathsf{M}(x, y))]$	Law 1 on (b)
(3)	$\sim [\mathsf{D}(v_1) \wedge \exists y(\mathsf{C}(y) \wedge \mathsf{M}(v_1, y))]$	U. I. on (2)
(4)	$\mathsf{D}(v_1) \to \sim \exists y(\mathsf{C}(y) \wedge \mathsf{M}(v_1, y))$	Cond. on (3)
(5)	$\mathsf{D}(v_1) \to \forall y \sim (\mathsf{C}(y) \wedge \mathsf{M}(v_1, y))$	Law 1 on (4)
(6)	$\mathsf{B}(v_1)$	Aux. Assumption
(8)	$\mathsf{I}(v_1)$	M.P. on $(1), (6)$
(9)	$\mathsf{I}(v_1) \to \mathsf{D}(v_1)$	U.I. on (c)
(10)	$\mathsf{D}(v_1)$	M.P. on $(9), (8)$
(11)	$\forall y \sim (\mathsf{C}(y) \wedge \mathsf{M}(v_1, y))$	M.P. on $(10), (5)$
(12)	$\sim (\mathsf{C}(v_2) \wedge \mathsf{M}(v_1, v_2))$	U.I. on (11)
(13)	$(\mathsf{C}(v_2) \to \sim \mathsf{M}(v_1, v_2))$	Cond. on (12)
(14)	$\forall y(\mathsf{C}(y) \to \sim \mathsf{M}(v_1, y))$	U.G. on (13)
(15)	$\mathsf{B}(v_1) \to \forall y(\mathsf{C}(y) \to \sim \mathsf{M}(v_1, y))$	C.P. on $(6) - (14)$
(16)	$\forall x(\mathsf{B}(x) \to \forall y(\mathsf{C}(y) \to \sim \mathsf{M}(x, y)))$	U.G. on (15)

Answering Questions

Claim	Logic
x, a question, y, a human	$\mathsf{Q}(x), \mathsf{H}(y)$
y attempted x , y answered x	$\mathsf{T}(y,x),\ \mathsf{A}(y,x)$
everyone attempted x	$\forall y(\mathsf{H}(y) \rightarrow \mathsf{T}(y,x))$
x is a question and everyone attempted x	$\forall y(\mathsf{H}(y) \rightarrow \mathsf{T}(y,x)) \wedge \mathsf{Q}(x)$
no one answered x	$\sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x))$
no one answered any x such that $\mathcal{P}(x)$	$\forall x(\boxed{\mathcal{P}(x)} \rightarrow \sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$
	$\sim \exists z(\mathsf{H}(z) \wedge \exists x(\mathcal{P}(x) \wedge \mathsf{A}(z,x)))$

No one answered any question that everyone attempted.

$$\mathcal{P}(x) = \forall y(\mathsf{H}(y) \rightarrow \mathsf{T}(y,x))$$

$$\forall x((\forall y(\mathsf{H}(y) \rightarrow \mathsf{T}(y,x)) \wedge \mathsf{Q}(x)) \rightarrow \sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$$

Scope issues

Three different claims (c is Richard's answer):

$$(a)\quad \forall x((\forall y(\mathsf{H}(y) \rightarrow \mathsf{T}(y,x)) \wedge \mathsf{Q}(x)) \rightarrow\sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$$

$$(b)\quad \forall x\forall y(((\mathsf{H}(y) \rightarrow \mathsf{T}(y,x)) \wedge \mathsf{Q}(x)) \rightarrow\sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$$

$$(c)\quad \forall x\forall y(((\mathsf{H}(y) \wedge \mathsf{T}(y,x)) \wedge \mathsf{Q}(x)) \rightarrow\sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$$

(b) : Every question (including attempted ones) went unanswered

$$(\mathsf{H}(q) \rightarrow \mathsf{T}(q,q)) \wedge \mathsf{Q}(q)$$

F F T

T T

T

(c) For every human y and question x such that y attempted x, x went unanswered $\Leftrightarrow$ Every attempted question went unanswered $\not\Leftrightarrow$ Every q. attempted by everyone did.

Equivalences I

(1)	$\sim \exists z(\mathsf{H}(z) \wedge \exists x(\mathcal{P}(x) \wedge \mathsf{A}(z,x)))$	
(2)	$\forall z \sim (\mathsf{H}(z) \wedge \exists x(\mathcal{P}(x) \wedge \mathsf{A}(z,x)))$	Law 1 (QNeg)
(3)	$\forall z \sim \exists x(\mathsf{H}(z) \wedge (\mathcal{P}(x) \wedge \mathsf{A}(z,x)))$	Law ?? (QMove)
(4)	$\forall z \sim\sim \forall x \sim (\mathsf{H}(z) \wedge (\mathcal{P}(x) \wedge \mathsf{A}(z,x)))$	Law 1 (QNeg)
(5)	$\forall z \forall x \sim (\mathsf{H}(z) \wedge (\mathcal{P}(x) \wedge \mathsf{A}(z,x)))$	Negation
(6)	$\forall z \forall x \sim (\mathcal{P}(x) \wedge \mathsf{H}(z) \wedge \mathsf{A}(z,x))$	Assoc
(7)	$\forall z \forall x \sim (\mathcal{P}(x) \wedge (\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$	Commut
(8)	$\forall z \forall x(\mathcal{P}(x) \rightarrow\sim (\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$	Cond.
(9)	$\forall z(\mathcal{P}(x) \rightarrow \forall x \sim (\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$	Law 9 (QMove)
(10)	$\forall x(\mathcal{P}(x) \rightarrow\sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$	Law 1 (QNeg)

Equivalences II

No one answered any question that everyone attempted.

$$\forall x((\forall y(\mathsf{H}(y) \rightarrow \mathsf{T}(y,x)) \wedge \mathsf{Q}(x)) \rightarrow \sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$$

Law 11 (QMove) $(\forall x)(\phi(x)) \rightarrow \psi \Leftrightarrow (\exists x)(\phi(x) \rightarrow \psi)$
provided x is not free in ψ

$$\forall x \exists y((\mathsf{H}(y) \rightarrow \mathsf{T}(y,x)) \wedge \mathsf{Q}(x)) \rightarrow \sim \exists z(\mathsf{H}(z) \wedge \mathsf{A}(z,x)))$$