

Logic
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1 Statement Logic

1.1 Laws for Truth Functional Connectives

1. Idempotent Laws

(a) $(P \vee P) \iff P$

(b) $(P \wedge P) \iff P$

2. Associative Laws

(a) $((P \vee Q) \vee R) \iff (P \vee (Q \vee R))$

(b) $((P \wedge Q) \wedge R) \iff (P \wedge (Q \wedge R))$

3. Commutative Laws

(a) $(P \vee Q) \iff (Q \vee P)$

(b) $(P \wedge Q) \iff (Q \wedge P)$

4. Distributive Laws

(a) $((P \vee Q) \wedge (P \vee R)) \iff (P \vee (Q \wedge R))$

(b) $((P \wedge Q) \vee (P \wedge R)) \iff (P \wedge (Q \vee R))$

5. Identity Laws

(a) $(P \vee F) \iff P$

(b) $(P \vee T) \iff T$

(c) $(P \wedge F) \iff F$

(d) $(P \wedge T) \iff P$

6. Complement Laws

(a) $(P \vee \neg P) \iff T$

(b) $\neg\neg P \iff P$

(c) $(P \wedge \neg P) \iff F$

7. De Morgan's Laws

(a) $\neg(P \vee Q) \iff (\neg Q \wedge \neg P)$

(b) $\neg(P \wedge Q) \iff (\neg Q \vee \neg P)$

Not in text but also useful

(c) $(P \vee Q) \iff \neg(\neg Q \wedge \neg P)$

(d) $(P \wedge Q) \iff \neg(\neg Q \vee \neg P)$

8. Conditional Laws

(a) $(P \rightarrow Q) \iff (\neg P \vee Q)$

(b) $(P \rightarrow Q) \iff (\neg Q \rightarrow \neg P)$

(c) $(P \rightarrow Q) \iff \neg(P \wedge \neg Q)$

9. Biconditional Laws

(a) $(P \iff Q) \iff ((P \rightarrow Q) \wedge (Q \rightarrow P))$

(c) $(P \iff Q) \iff ((\neg Q \wedge \neg P) \wedge (P \wedge Q))$

1.2 Statement Logic Rules of Inference

<u>Name and Abbr.</u>	<u>Form</u>	<u>Example</u>
Modus Ponens (M. T.)	$\frac{P \rightarrow Q}{P}$ <hr/> Q	If Fred sang loud, Mary danced. Fred sang loud. <hr/> Mary danced.
Modus Tollens (M. T.)	$\frac{P \rightarrow Q}{\neg Q}$ <hr/> $\neg P$	If Fred sang loud, Mary danced. Mary didn't dance. <hr/> Fred didn't sing loud.
Hypothetical Syllogism (H. S.)	$\frac{P \rightarrow Q}{Q \rightarrow R}$ <hr/> $P \rightarrow R$	If 3 is bigger than 2 then 3 is bigger than 1. If 3 is bigger than 1 then 3 is bigger than 0. <hr/> If 3 is bigger than 2, then 3 is bigger than 0.
Disjunctive Syllogism (D. S.)	$\frac{P \vee Q}{\neg P}$ <hr/> Q	Either the bathroom is here or the bathroom is upstairs. The bathroom is not here. <hr/> The bathroom is upstairs.
Simplification (Simp.)	$\frac{P \wedge Q}{P}$	Apples are sweet and radishes are bitter. <hr/> Apples are sweet.
Conjunction (Conj.)	$\frac{P}{Q}$ <hr/> $P \wedge Q$	Apples are sweet. Radishes are bitter. <hr/> Apples are sweet and radishes are bitter.
Addition (Add.)	$\frac{P}{P \vee Q}$	Apples are sweet. <hr/> Apples are sweet or radishes are bitter.