Midterm

Linguistics 570: Mathematical Linguistics

October 24, 2011

1 Introduction

Throughout this exam, assume

$$\text{N} = \{0,\ 1,\ 2,\ 3,\ \ldots\}$$
$$\text{Z} = \{\ldots,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ \ldots\}$$

Please do this exam without collaboration. You are free to ask me as many questions as you like via email and at office hours. And I am free to answer as many as I like. The exam is due October 31.

2 Functions, Relations, Sets

In this section you are asked some questions about the successor function of arithmetic and the successor function of arithmetic mod 7.

2.1 Sets and functions

2.1 Sets can be specified in four ways, (i) by predicates, (ii) by recursive rules, (iii) by listing, and (iv) by name (some sets like the Beatles and The Empty Set have names). But some sets can be defined by listing and predicates only, and can't be defined by recursive rule; and some sets can be defined by recursive rules and predicates only, and can't be defined by listing. Finally some sets can't be defined at all. Each of (a)-(g) is an attempt to define a set. Do the following: (a) Determine whether this is a valid definition of a set; remember, a valid definition of a set doesn't have to be elegant; it just has to uniquely and completely specify a set; (b) Identify the type of definition (i), (ii), (iii), or (iv); (c) If a set is defined by a predicate, give an alternative recursive definition if possible, and if it is not possible, explain why; (d) If a set is defined by recursive rule, give an alternative definition by predicate; (e) For each set, determine whether the set can be defined by listing. If it can, say so; if it can't explain why. You do not actually have to give the list; (f) For each set, if the set has a name, give the name.

(a) $\mathrm{A} = \{x \mid x \text{ is an odd integer greater than 1 and less than 12.}\}$

(b) A = { Raiders of the lost Ark, Peter Pan
Jaws, Close Encounters of the Third Kind}

(c)

$$\mathrm{A} = \{x \mid x \text{ is a page in our text book }\}$$

Here by pages I mean types, not tokens. Page tokens are what copies of a book have; page types are what those tokens are instances of. That is, Eric owns a copy of our text book (I hope) and so does Adam (I hope) and these copies are distinct objects (let's assume) with distinct physical subparts called pages. These subparts are page tokens, In particular both copies have page tokens numbered 337 (a page in Chapter 13). The page type of which both tokens are an instance is what I mean by "page" in the definition of A. There are thus $664 + 22 + 2$ pages in our text book (664 pages with Arabic numerals, 22 pages of front matter, and 2 pages of back matter) and not some much larger number that depends on the exact number of copies printed.

(d) $\mathrm{A} = \{x \mid x \notin x\}$

(e) $\mathrm{A} = \{x \mid x \text{ is an odd number evenly divisible by two.}\}$

(f) i. $1 \in \mathrm{A}$

ii. If $x \in \mathrm{A}$, then $x + 2 \in \mathrm{A}$.

iii. Nothing else is in A.

(g) $\mathrm{A} = \{2^n \mid n \text{ is a positive integer}\}$

2.2 Let F be a function from the set of pages of our textbook as defined in 2.1.c to the natural numbers.

$$\mathrm{F} : \ \mathrm{A} \rightarrow \mathrm{N}$$

Assume the cardinality of the range of F is 688. For the following claims determine whether they must be true, can't be true, or may be true and may be false. Justify your choice.

(a) F is 1-1.

(b) F is onto.

(c) The domain of F is equal to A.

(d) F(the page numbered 337) $>$ F(the page numbered 336).

(e) F^{-1} is a function.

(f) $(F^{-1})^{-1} = F$.

(g) Given what we know about F so far, there are an infinite number of possible functions that F could be.

2.2 Set operations and relations

In this section you are asked to compute some set operations and some relations.

Assume the following sets with the universe of discourse $\{1, 2, 3\}$:

$$\begin{aligned} A &= \{1,\ 2\} \\ B &= \{2,\ 3\} \\ C &= \{1,\ 3\} \end{aligned}$$

Compute the following. Show your work:

2.1 $(A \times B) \cap (A \times A)$

2.2 $(A - B) \cap (B - A)$

2.3 Simplify the following expression using De Morgan's Law:

$$(A' \cup B')'$$

Now, compute it.

2.4 $\{1, 2, 3\} \cap \{x \mid x \text{ is a perfect square less than 27}\}$

2.3 Equivalence Relations

Consider the relation R in the set Z defined as follows:

$$R = \{\langle a, b\rangle \mid a - b \text{ is evenly divisible by 3}\}$$

A number x is "evenly divisible by 3" if and if the remainder when x is divided by 3 is 0. Note: the definition of "evenly divisible" is the same for 0, negative and positive numbers, so 0, 3, -6 and -12 are evenly divisible by 3 and -7 and -10 are not.

Answer the following questions and justify your answers:

2.1 Is R reflexive?

2.2 Is R symmetric?

2.3 Which of the following pairs are in R?

$$\begin{array}{ccc} \langle 3,7\rangle & \langle 3,8\rangle & \langle 3,9\rangle \\ \langle 4,7\rangle & \langle 4,8\rangle & \langle 4,9\rangle \\ \langle 4,10\rangle & \langle 7,10\rangle & \langle 0,3\rangle \end{array}$$

2.4 Is R transitive? It may help to consider the following. Pick a member of Z and call it x; when we divide x by 3 we get a quotient q and a remainder r; that is,

$$x = 3q + r.$$

For example, for $x = 7$, $q = 2$ and $r = 1$. For $x = 12$, $q = 4$ and $r = 0$. For a number to be evenly divisible by 3 means $r = 0$. Now consider subtraction:

$$\begin{array}{lcl} x & = & 3q + r \\ y & = & 3q' + r' \\ x - y & = & 3q + r - (3q' - r') \\ & = & 3q - 3q' + r - r' \\ & = & 3(q - q') + (r - r') \end{array}$$

What the last line tells us is that in order for $x - y$ to be evenly divisible by 3, $r - r'$ has to be equal to 0, or in other words $r = r'$.

2.5 Is R an equivalence relation?

2.6 If R is an equivalence relation, give a predicate definition of each equivalence class; if it is not, give a recursive definition of R.

3 Statement Logic

Translate the following into statement logic:

3.1 Only if the economy rebounds will the unemployment rate fall.

3.2 The political climate in Washington will improve when we have a 3-party system.

3.3 The Democrats will lose unless Obama is not nominated.

3.4 Neither Mitt Romney nor Rick Perry can win in November.

3.5 Alice or Bonnie loves Charlie and Fred.

3.1 Other matters of statement logic

3.1 Prove using the natural deduction system of chapter 6

$$\begin{array}{l} \sim (p \rightarrow q) \\ r \rightarrow q \\ r \vee s \\ \underline{(s \vee t) \rightarrow u} \\ u \end{array}$$

4 Predicate logic

4.1 All horses are quadripeds but some quadripeds are not horses.

4.2 No ducks are amphibious.

4.3 Only Rosicrucians experience complete happiness.

4.4 Everything I like is immoral, illegal, or fattening.

4.5 Prove the validity of the following argument. You may translate *my aunt* using a constant (a); so the translation of *my aunt walks* would be W(a). Similarly for *the parity principle* (p).

> No linguist believes in the parity principle. Everyone believes in the parity principle or is a behaviorist. Every dietician renounces behaviorism. My aunt is a dietician. Therefore, there is someone who is neither a linguist nor a behaviorist.

4.6 Prove the validity of the following argument.

> All cabdrivers and headwaiters are surly and churlish. Therefore, all cabdrivers are surly.

If you're having trouble with this proof, go back and check your translation of the premise carefully. Have you really got it right?