Finite-State Transducer Composition
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1 FSA Definition

We define an FSA (Finite-State Automaton) to be a 5-tuple:

$$\langle \mathrm{Q}, \Sigma, \delta, q_0, \mathrm{F} \rangle$$

with

Q : a set of states
Σ : an input alphabet
δ : a transition function, (from $\Sigma \times$ Q to Q)
q_0 : an initial state
F : a set of final states

2 Transducer Definition

A transducer is defined with respect to a set of pairs π, which is the analogue of the alphabet Σ. every FSA is defined with. The set π is a set of *feasible pairs* of elements from some alphabet Σ_1 and Σ_2, the input and output alphabets.

$$\pi = \{a : b \mid a \in \Sigma_1, b \in \Sigma_2, \text{ and the pair } a : b \text{ occurs in the language we're describing.}\}$$

Example: in the set of feasible pairs in the language we've been using for translating morphological rules into transducers

$$\hat{} : \epsilon \in \pi$$

Moreover, ϵ is the only surface realization ˆ is paired with in π. This means perfectly good pairs from $\Sigma \times \Sigma$ like ˆ:p, ˆ:q, and ˆ:# are not in π.

We define an FST (Finite-State Transducer) to be a 5-tuple:

$$\langle \mathrm{Q}, \pi, \delta, q_0, \mathrm{F} \rangle$$

with

$$\begin{array}{l} \mathrm{Q}: \text{ a set of states} \\ \Sigma_1: \text{ an input alphabet} \\ \Sigma_2: \text{ an output alphabet} \\ \pi \subseteq \Sigma_1 \times \Sigma_2 \\ \delta: \text{ a transition function, (from } \pi \times \mathrm{Q} \text{ to } \mathrm{Q}\text{)} \\ q_0: \text{ an initial state} \\ \mathrm{F}: \text{ a set of final states} \end{array}$$

3 Definition of Composition of two transducers

We write the **composition** of two transducers T_1 and T_2 as $\mathrm{T}_1 \circ \mathrm{T}_2$.

Let

$$\begin{array}{l} \mathrm{T}_1 = \langle \mathrm{Q}_1, \pi_1, \delta_1, q1_0, \mathrm{F}_1 \rangle \\ \mathrm{T}_2 = \langle \mathrm{Q}_2, \pi_2, \delta_2, q2_0, \mathrm{F}_2 \rangle \end{array}$$

where Σ_1, Σ_2, and Σ_3 are called the input, the intermediate and the output alphabets respectively, and

$$\begin{array}{l} \pi_1 \subseteq \Sigma_1 \times \Sigma_2 \\ \pi_2 \subseteq \Sigma_2 \times \Sigma_3 \end{array}$$

Then $\mathrm{T}_1 \circ \mathrm{T}_2$ is defined as:

$$\langle \mathrm{Q}_1 \times \mathrm{Q}_2, \pi_\circ, \delta_\circ, \langle q1_0, q2_0 \rangle, \mathrm{F}_1 \times \mathrm{F}_2 \rangle$$

where

$$\begin{array}{l} \pi_\circ = \{a : b \mid \; a \in \Sigma_1, b \in \Sigma_3 \text{ and there is some } x \in \Sigma_2 \text{ such that} \\ \qquad\qquad\qquad a : x \in \pi_1 \text{ and } x : b \in \pi_2 \} \text{ and} \\ \delta_\circ(\langle q_i, q_j \rangle, a : b) = \langle q_k, q_l \rangle \text{ if and only if} \\ \text{ there is some } x \in \Sigma_2 \text{ such that } \; \delta_1(q_i, a : x) = q_k \text{ and} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \delta_2(q_j, x : b) = q_l \end{array}$$