Introduction to Computational Linguisyics

http://www-rohan.sdsu.edu/~gawron/compling

Naive Bayes

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Overview

1. Sense disambiguation
2. Naive Bayes
3. NB theory

An ambiguous word

The noun "bass" has 8 senses in WordNet.

1. bass^{1} - (the lowest part of the musical range)
2. bass^{2}, bass part^{1} - (the lowest part in polyphonic music)
3. bass^{3}, basso^{1} - (an adult male singer with the lowest voice)
4. sea bass^{1}, bass^{4} - (the lean flesh of a saltwater fish of the family Serranidae)
5. freshwater bass^{1}, bass^{5} - (any of various North American freshwater fish with lean flesh (especially of the genus Micropterus))
6. bass^{6}, bass voice^{1}, basso^{2} - (the lowest adult male singing voice)
7. bass^{7} - (the member with the lowest range of a family of musical instruments)
8. bass^{8} - (nontechnical name for any of numerous edible marine and freshwater spiny-finned fishes)

The adjective "bass" has 1 sense in WordNet.

1. bass^{1}, deep^{6} - (having or denoting a low vocal or instrumental range) *"a deep voice"; "a bass voice is lower than a baritone voice"; "a bass clarinet"*

Examples

- He plays the bass well.
- He caught a huge bass.
- They served fried bass for lunch.
- She carried her bass clarinet into class.
- Charlie is the bass in a barbershop quartet.

The Naive Bayes classifier

- The Naive Bayes classifier is a probabilistic classifier.
- We will estimate the probability of a word like *bass* in a document d being a use of word sense s as follows:

$$P(s \mid d) \propto P(s) \prod_{1 \leq k \leq n_d} P(t_k \mid s)$$

  where n_d is the number of **context features** in document d, taken from a set of context features (words) for disambiguating *bass*.
- $P(t_k|s)$ is the conditional probability of context feature t_k occurring in the same document with sense s.
- $P(t_k|s)$ as a measure of how much evidence t_k contributes that s is the correct sense.
- $P(s)$ is the prior probability of sense s.
- If a context does not provide clear evidence for one sense vs. another, we choose the s with highest $P(s)$.

Maximum a posteriori class

- Our goal in Naive Bayes classification is to find the "best" class.
- The best class is the most likely or maximum a posteriori (MAP) sense s_{map}:

$$s_{\text{map}} = \arg\max_{s \in \mathbb{S}} \hat{P}(s|w) = \arg\max_{s \in \mathbb{S}} \hat{P}(s) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|s)$$

- Note that

$$P(s) \prod_{1 \leq k \leq n_d} P(t_k \mid s) \tag{1}$$

is **not** the conditional probability, $P(s \mid d)$. What it actually is is the joint probability of the sense and the document.

$$P(s,\, d) = P(s) \prod_{1 \leq k \leq n_d} P(t_k \mid s)$$

- For now, the conditional probability is not needed, because the joint probability, which is easier to estimate, is **proportional** to it.

Taking the log

- Multiplying lots of small probabilities can result in floating point underflow.
- Since $\log(xy) = \log(x) + \log(y)$, we can sum log probabilities instead of multiplying probabilities.
- Since log is a monotonic function, the class with the highest score does not change.
- So what we usually compute in practice is:

$$c_{\text{map}} = \arg\max_{s \in \mathbb{C}} [\log \hat{P}(s) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|s)]$$

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  - The sum of log prior and term weights is then a measure of how much evidence there is for the document being in the class.
  - We select the class with the most evidence.

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- We've made a Naive Bayes independence assumption here:

$$\hat{P}(t_j, t_k|s) \;\; = \;\; \hat{P}(t_j|s)\hat{P}(t_k|s, t_j) \;\; = \;\; \hat{P}(t_j|s)\hat{P}(t_k|s)$$

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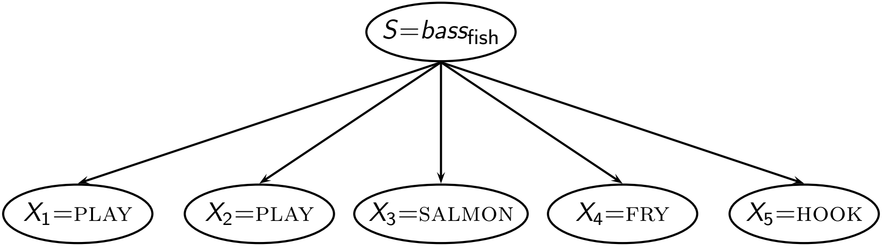

$$
\begin{aligned}
P(bass_{\text{fish}}|d) \quad \propto \quad & P(bass_{\text{fish}}) \cdot P(\text{PLAY}|bass_{\text{fish}}) \cdot P(\text{PLAY}|bass_{\text{fish}}) \\
& \cdot P(\text{SALMON}|bass_{\text{fish}}) \cdot P(\text{FRY}|bass_{\text{fish}}) \cdot P(\text{HOOK}|bass_{\text{fish}})
\end{aligned}
$$

- If HOOK never occurs with sense $bass_{\text{fish}}$ in the training set:

$$\hat{P}(\text{HOOK}|bass_{\text{fish}}) = \frac{T_{bass_{\text{fish}},\text{HOOK}}}{\sum_{t' \in V} T_{bass_{\text{fish}},t'}} = \frac{0}{\sum_{t' \in V} T_{bass_{\text{fish}},t'}} = 0$$

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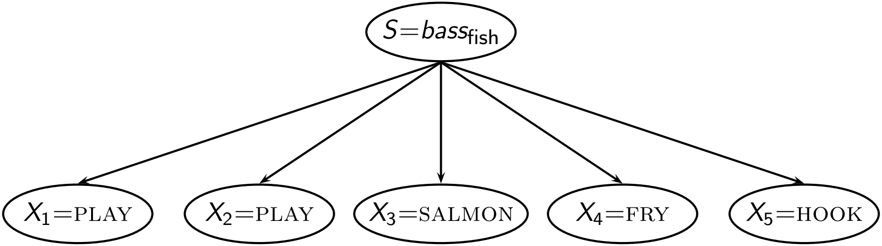

$$P(bass_{\text{fish}}|d) \propto P(bass_{\text{fish}}) \cdot P(\text{PLAY}|bass_{\text{fish}}) \cdot P(\text{PLAY}|bass_{\text{fish}}) \cdot P(\text{SALMON}|bass_{\text{fish}}) \cdot P(\text{FRY}|bass_{\text{fish}}) \cdot P(\text{HOOK}|bass_{\text{fish}})$$

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The problem with maximum likelihood estimates: Zeros (cont)

- If there were no occurrences of HOOK in documents in class $bass_{\text{fish}}$, we'd get a zero estimate:

$$\hat{P}(\text{HOOK}|bass_{\text{fish}}) = \frac{T_{bass_{\text{fish}},\text{HOOK}}}{\sum_{t' \in V} T_{bass_{\text{fish}},t'}} = 0$$

- $\rightarrow$ We will get $P(bass_{\text{fish}}|w) = 0$ for any document that contains hook! No matter how much positive evidence there is for one of the senses.
- Zero probabilities cannot be conditioned away.

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- Now: Add one to each count to avoid zeros:

$$\hat{P}_{sm}(t|s) = \frac{T_{st} + 1}{\sum_{t' \in V_w} (T_{ct'} + 1)} = \frac{T_{st} + 1}{(\sum_{t' \in V} T_{st'}) + |V_w|}$$

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- V_w is the set of context features for w (*bass*, the word we are disambiguating), and $|V_w|$ is the number of such features.

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- Estimate parameters from the training corpus using add-one smoothing
- For a new document, for each class, compute sum of (i) log of prior and (ii) logs of conditional probabilities of the terms
- Assign the document to the class with the largest score

Naive Bayes: Training

TrainMultinomialNB($\mathbb{S}, \mathbb{D}, w$)

$V \leftarrow$ ExtractFeatures($\mathbb{D}, w$)
$N \leftarrow$ CountOccurrences($\mathbb{D}, w$)
for each $s \in \mathbb{S}$
do $N_s \leftarrow$ CountOccurrencesOfSense($\mathbb{D}, s$)
    $prior[s] \leftarrow N_s / N$
    $text_s \leftarrow$ ConcatTextOfAllContextsWithSense($\mathbb{D}, s$)
    for each $t \in V$
    do $T_{st} \leftarrow$ CountTokensOfTerm($text_s, t$)
    for each $t \in V$
    do $condprob[t][s] \leftarrow \frac{T_{st}+1}{\sum_{t'}(T_{st'}+1)}$
return $V, prior, condprob$

Naive Bayes: Testing

ApplyMultinomialNB($\mathbb{S}, V, prior, condprob, d$)

$W \leftarrow$ ExtractFeatureTokensFromDoc(V, d)
for each $s \in \mathbb{S}$
do $score[c] \leftarrow \log prior[s]$
    for each $t \in W$
    do $score[s] += \log condprob[t][s]$
return $\arg\max_{s \in \mathbb{S}} score[s]$

Exercise

For our feature set, we often choose the n most frequent content words in our document set, retaining duplicates, leaving out w (*bass*). Here we choose *fry, play, clarinet, salmon, hook*, and *guitar*. Note that *guitar* occurs in neither the test set nor training set. Assume the context window for training

	docID	words in context	in $s = bass_{\text{fish}}$?
training set	1	fry fry	yes
	2	play play clarinet	no
	3	salmon fry	yes
	4	play	yes
test set	5	play play fry hook play play play play play	?

- Estimate parameters of Naive Bayes classifier
- Classify test document

Unsmoothed Example: Parameter estimates

Priors: $\hat{P}(s) = 3/4$ and $\hat{P}(\overline{s}) = 1/4$
Conditional probabilities:

$$
\begin{aligned}
\hat{P}(\textsc{fry}|s) &= 3/5 \\
\hat{P}(\textsc{salmon}|s) = \hat{P}(\textsc{play}|s) &= 1/5 \\
\hat{P}(\textsc{hook}|s) &= 0/5 = 0 \\
\hat{P}(\textsc{clarinet}|\overline{s}) &= 1/3 \\
\hat{P}(\textsc{play}|\overline{s}) &= 2/3
\end{aligned}
$$

The denominators are 5 and 3 because the lengths of $text_s$ and $text_{\overline{s}}$ are 5 and 3, respectively.

Smoothing Example: Parameter estimates

Priors: $\hat{P}(s) = 3/4$ and $\hat{P}(\overline{s}) = 1/4$
Conditional probabilities:

$$\begin{aligned}
\hat{P}_{sm}(\text{FRY}|s) &= (3+1)/(5+6) = 4/11 \\
\hat{P}_{sm}(\text{SALMON}|s) = \hat{P}_{sm}(\text{PLAY}|s) &= (1+1)/(5+6) = 2/11 \\
\hat{P}_{sm}(\text{HOOK}|s) &= (0+1)/(5+6) = 1/11 \\
\hat{P}_{sm}(\text{CLARINET}|\overline{s}) &= (1+1)/(3+6) = 2/9 \\
\hat{P}_{sm}(\text{PLAY}|\overline{s}) &= (2+1)/(3+6) = 3/9 = 1/3
\end{aligned}$$

The denominators are $(5+6)$ and $(3+6)$ because the lengths of $text_s$ and $text_{\overline{s}}$ are 5 and 3, respectively, and because the constant $|V_w|$ is 6 as the feature set consists of six terms.

Example: Classification

$$
\begin{aligned}
\hat{P}_{sm}(s|d_5) &\propto 3/4 \cdot (2/11)^7 \cdot 4/11 \cdot 1/11 \approx 1.643 * 10^{-7} \\
\hat{P}_{sm}(\overline{s}|d_5) &\propto 1/4 \cdot (1/3)^7 \cdot 1/9 \cdot 1/9 \approx 2.389 * 10^{-7}
\end{aligned}
$$

Thus, the classifier assigns the test document to $s = \overline{bass_{\text{fish}}}$.
The reason for this classification decision is that the seven occurrences of the negative indicator PLAY in d_5 outweigh the occurrence the positive indicator FRY.
Note: the terms to the right of $\propto$ are what we were referring to as **joint probabilities** in first few slides.

Normalizing Joint probabilities

To get from joint probability $\hat{P}(s_1, t_{1,n_d})$ to the conditional probability $\hat{P}(s_1 \mid t_{1,n_d})$ we must normalize:

$$
\begin{array}{ll}
\text{(a)} & \hat{P}(s_1, t_{1,n_d}) = \hat{P}(s) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|s_1) = 0.00099 \\
\text{(b)} & \hat{P}(s_1|t_{1,n_d}) = \hat{P}(s_1, t_{1,n_d}) / \hat{P}(d) \\
\text{(c)} & \hat{P}(s_2, t_{1,n_d}) = \hat{P}(s) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|s_1) = 0.00001 \\
\text{(d)} & \hat{P}(s_2|t_{1,n_d}) = \hat{P}(s_2, t_{1,n_d}) / \hat{P}(d) \\
\text{(e)} & \hat{P}(s_1|t_{1,n_d}) + \hat{P}(s_2|t_{1,n_d}) = \frac{\hat{P}(s_1, t_{1,n_d}) + \hat{P}(s_2, t_{1,n_d})}{\hat{P}(d)} = 1.0 \\
\text{(f)} & \hat{P}(s_1, t_{1,n_d}) + \hat{P}(s_2, t_{1,n_d}) = \hat{P}(d) = 0.0001 \\
\text{(g)} & \hat{P}(s_1|t_{1,n_d}) = 0.99, \hat{P}(s_2|t_{1,n_d}) = 0.01.
\end{array}
$$

How to normalize

$$\hat{P}(s_1|t_{1,n_d}) = \frac{\hat{P}(s_1, t_{1,n_d})}{\sum_i \hat{P}(s_i, t_{1,n_d})}$$

In our example there are only two senses, so:

$$\hat{P}(s_1|t_{1,n_d}) = \frac{\hat{P}(s_1, t_{1,n_d})}{\hat{P}(s_1, t_{1,n_d}) + \hat{P}(s_2, t_{1,n_d})} = \frac{.00099}{.00099 + .00001} = \frac{.00099}{.001} = .99.$$

Since

$$\hat{P})(t_{1,n_d}) = \sum_{s_i} \hat{P}(s_i, t_{1,n_d}) = \hat{P}(s_1, t_{1,n_d}) + \hat{P}(s_2, t_{1,n_d})$$

this is just a rewrite of our definition of conditional probability:

$$\hat{P}(s_1|t_{1,n_d}) = \frac{\hat{P}(s_1, t_{1,n_d})}{\hat{P}(t_{1,n_d})}$$

Time complexity of Naive Bayes

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mode	time complexity
training	$\Theta(\lvert\mathbb{D}\rvert L_{\text{ave}} + \lvert\mathbb{S}\rvert\lvert V\rvert)$
testing	$\Theta(L_{\text{a}} + \lvert\mathbb{S}\rvert M_{\text{a}}) = \Theta(\lvert\mathbb{S}\rvert M_{\text{a}})$

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- Generally: $|\mathbb{S}||V| < |\mathbb{D}|L_{\mathrm{ave}}$

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- Generally: $\lvert\mathbb{S}\rvert\lvert V\rvert < \lvert\mathbb{D}\rvert L_{\mathrm{ave}}$
- Test time is also linear (in the length of the test document).
- Thus: Naive Bayes is linear in the size of the training set (training) and the test document (testing). This is optimal.

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- We will formally derive the classification rule . . .
- . . . and state the assumptions we make in that derivation explicitly.

Derivation of Naive Bayes rule

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We want to find the class that is most likely given the document:

$$c_{\text{map}} = \underset{s \in \mathbb{C}}{\arg\max}\ P(s|d)$$

Apply Bayes rule $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$:

$$c_{\text{map}} = \underset{s \in \mathbb{C}}{\arg\max}\ \frac{P(d|s)P(s)}{P(d)}$$

Drop denominator since $P(d)$ is the same for all classes:

$$c_{\text{map}} = \underset{s \in \mathbb{C}}{\arg\max}\ P(d|s)P(s)$$

Too many parameters / sparseness

$$
\begin{aligned}
c_{\text{map}} &= \underset{s \in \mathbb{C}}{\arg\max}\; P(d|s)P(s) \\
&= \underset{s \in \mathbb{C}}{\arg\max}\, P(\langle t_1, \ldots, t_k, \ldots, t_{n_d} \rangle | s)P(s)
\end{aligned}
$$

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- There are too many parameters $P(\langle t_1, \ldots, t_k, \ldots, t_{n_d} \rangle | s)$, one for each unique combination of a class and a sequence of words.

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- There are too many parameters $P(\langle t_1, \ldots, t_k, \ldots, t_{n_d} \rangle | s)$, one for each unique combination of a class and a sequence of words.
- We would need a very, very large number of training examples to estimate that many parameters.

Too many parameters / sparseness

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- This is the problem of data sparseness.

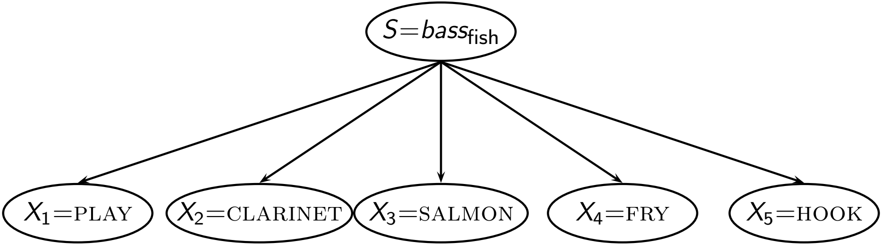
Naive Bayes conditional independence assumption

To reduce the number of parameters to a manageable size, we make the Naive Bayes conditional independence assumption:

$$P(d|s) = P(\langle t_1, \ldots, t_{n_d} \rangle | s) = \prod_{1 \leq k \leq n_d} P(X_k = t_k | s)$$

We assume that the probability of observing the conjunction of attributes is equal to the product of the individual probabilities $P(X_k = t_k|s)$. Recall from earlier the estimates for these priors and conditional probabilities: $\hat{P}(s) = \frac{N_c}{N}$ and $\hat{P}(t|c) = \frac{T_{ct}+1}{(\sum_{t' \in V} T_{ct'})+B}$

Generative Model



$P(s|d) \propto P(s) \prod_{1 \leq k \leq |n_d|} P(t_k|c)$

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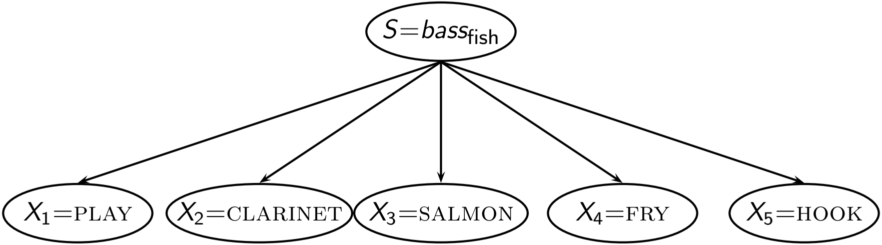

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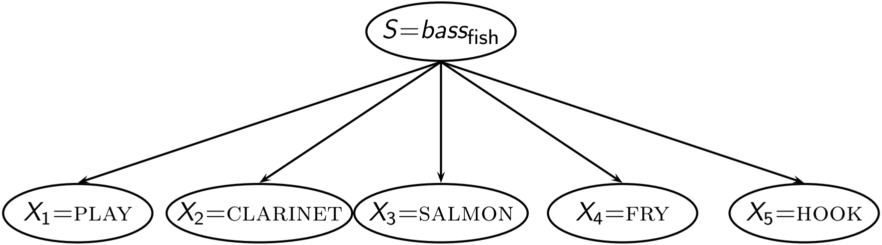

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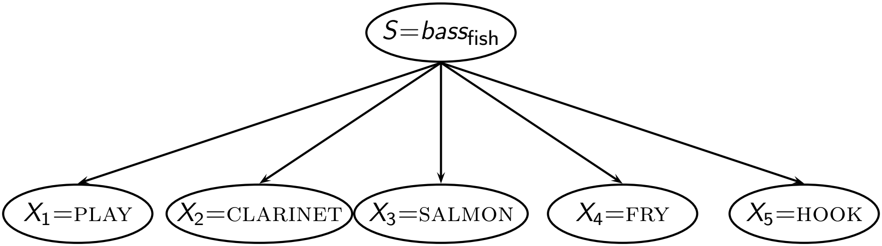

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- Generate a sense with probability $P(s)$
- Generate each of the context words, conditional on the sense, but independent of each other, with probability $P(t_k|s)$
- To classify docs, we "simulate" the generative process and find the sense that is most likely to have generated the doc.

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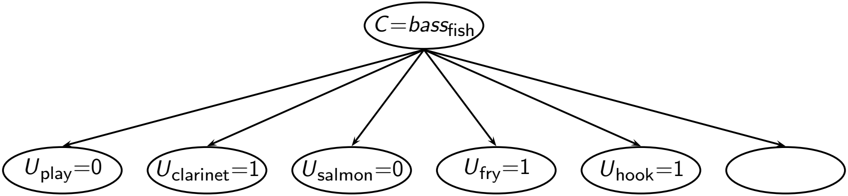
- For example, for a token of *bass* using the sense $bass_{\text{fish}}$, the probability of generating FRY in the first position in the document is the same as generating it in the last position.
- The two independence assumptions amount to the bag of words model (information retrieval: order of words in documents does not matter).

A different Naive Bayes model: Bernoulli model



multinomial $\quad c_{map} = \arg\max_{s \in \mathbb{C}} \left[\log \hat{P}(s) + \sum_{1 \leq k \leq |n_d|} \log \hat{P}(t_k|s)\right]$

bernoulli $\quad c_{map} = \arg\max_{s \in \mathbb{C}} \left[\log \hat{P}(s) + \sum_{1 \leq k \leq |V_w|} \log \hat{P}(t_k|s)\right]$

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- Exercise
  - Examples for why conditional independence assumption is not really true?
  - Examples for why positional independence assumption is not really true?
- How can Naive Bayes work if it makes such inappropriate assumptions?

Why does Naive Bayes work?

- Naive Bayes can work well even though conditional independence assumptions are badly violated.
- Example:

	s_1	s_2	sense selected
true probability $P(s\|d)$	0.6	0.4	s_1
$\hat{P}(s)\prod_{1\leq k\leq n_d}\hat{P}(t_k\|s)$	0.00099	0.00001	
NB estimate $\hat{P}(s\|d)$	0.99	0.01	s_1

Illustration

- Suppose the featues are NOT independent, causing, say, underestimation $\mathrm{P}(s_2 \mid d) = 0.01$).

  $$\mathrm{P}(s_2|t_k, t_j) \quad > \quad \hat{P}(s_1|t_k)\hat{P}(s_2|t_j)$$

  Then $\mathrm{P}(s_1|t_k, t_j)$ will be overestimated (0.99):

  $$\mathrm{P}(s_1|t_k, t_j) \quad < \quad \hat{P}(s_1|t_k)\hat{P}(s_1|t_j)$$

  As long as NB overestimates the larger prob, it will still make correct classification decisions. Even if NB overestimates the smaller prob, the decision *might* still be right.
- Classification is about predicting the correct class and not about accurately estimating probabilities.
- Correct estimation $\Rightarrow$ accurate prediction.
- But not vice versa!

Naive Bayes is not so naive

- Naive Bayes has won some bakeoffs (e.g., KDD-CUP 97)
- More robust to nonrelevant features than some more complex learning methods
- More robust to concept drift (changing of definition of class over time) than some more complex learning methods
- Better than methods like decision trees when we have many equally important features
- A good dependable baseline for text classification (but not the best)
- Optimal if independence assumptions hold (never true for text, but true for some domains)
- Very fast
- Low storage requirements