HMMs: Jason Eisner's Ice Cream HMM

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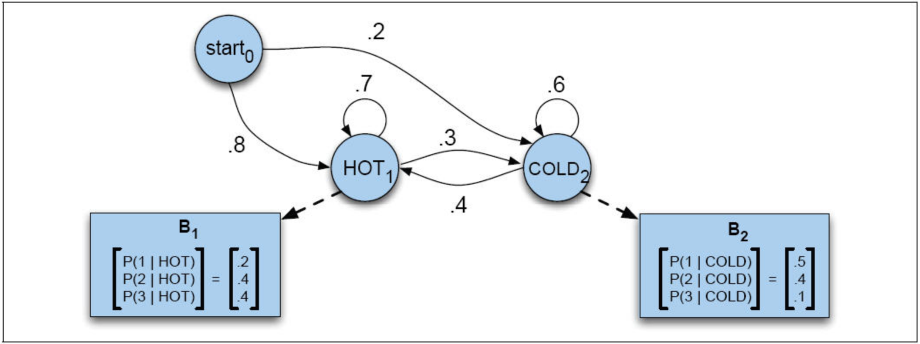
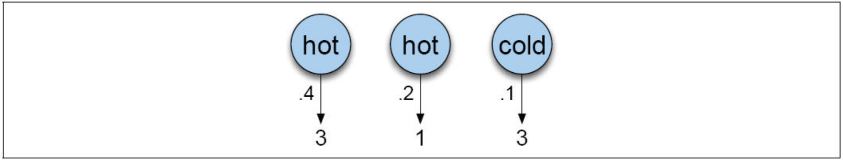
Outline

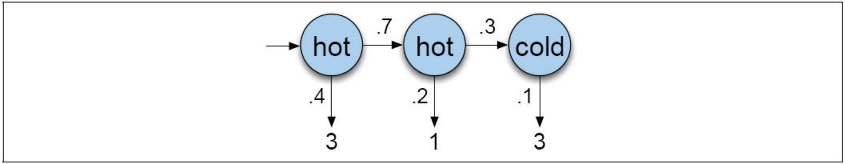
The Ice Cream HMM



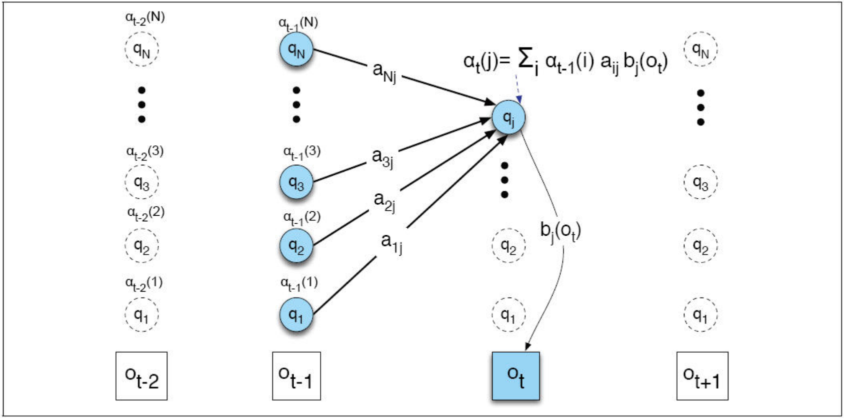
Observation Probs



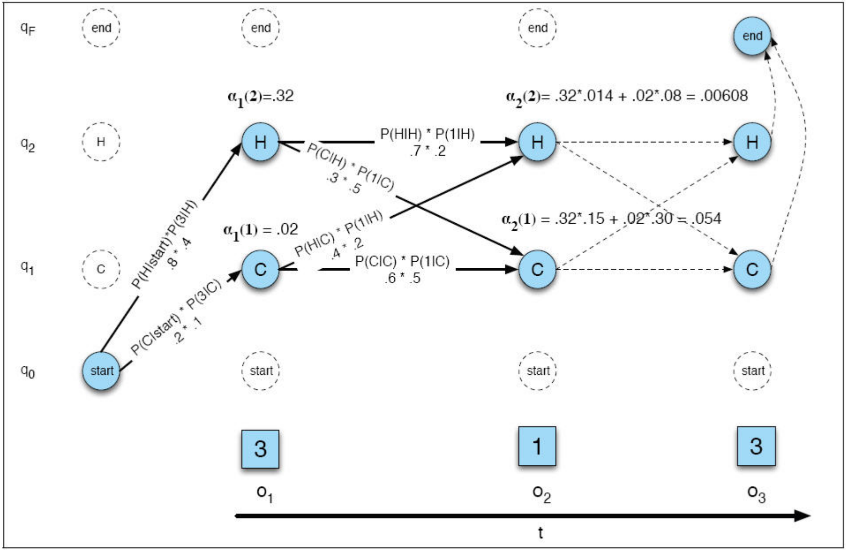
Transition Probs



Forward Prob ($\alpha(q_i)$)



The Trellis



q_F
q_2
q_1
q_0
end
end
end
end
H
C
start
α_1(2)=.32
α_1(1) = .02
α_2(2)= .32*.014 + .02*.08 = .00608
α_2(1) = .32*.15 + .02*.30 = .054
P(H|start)*P(3|H)
.8 * .4
P(C|start) * P(3|C)
.2 * .1
P(H|H) * P(1|H)
.7 * .2
P(C|H) * P(1|C)
.3 * .5
P(H|C) * P(1|H)
.4 * .2
P(C|C) * P(1|C)
.6 * .5
H
H
H
C
C
C
start
start
start
3
1
3
o_1
o_2
o_3
t

Forward Prob Algorithm

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*

for each state s **from** 1 **to** N **do** ; initialization step

$forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$

for each time step t **from** 2 **to** T **do** ; recursion step

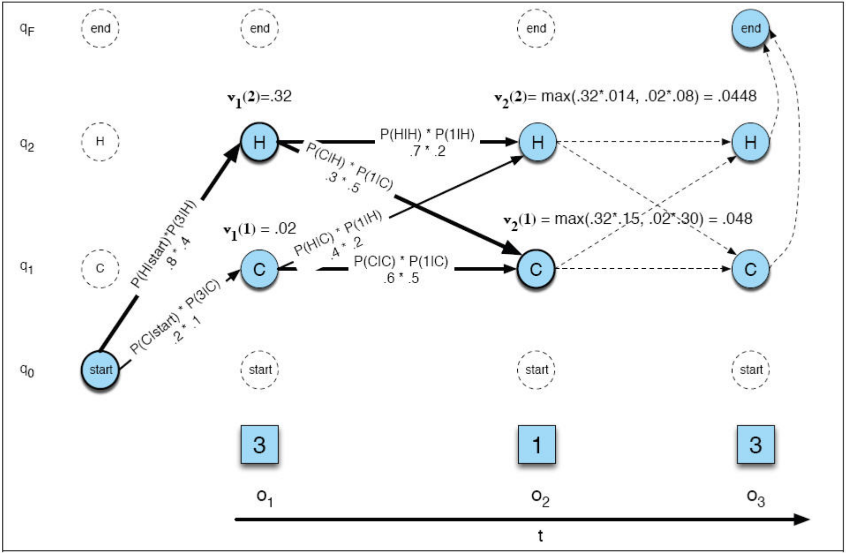
for each state s **from** 1 **to** N **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,T] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step

return $forward[q_F,T]$

Viterbi Computation



Viterbi Algorithm

function VITERBI(*observations* of len T, *state-graph* of len N) **returns** *best-path*

create a path probability matrix *viterbi[N+2,T]*

for each state s **from** 1 **to** N **do** ; initialization step

$viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$

$backpointer[s,1] \leftarrow 0$

for each time step t **from** 2 **to** T **do** ; recursion step

for each state s **from** 1 **to** N **do**

$$viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$$

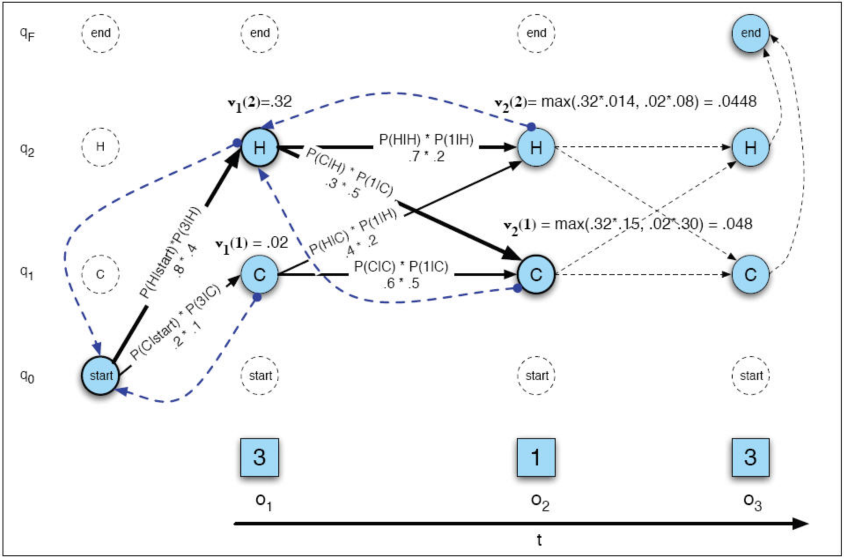
$$backpointer[s,t] \leftarrow \operatorname*{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$$

$$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step

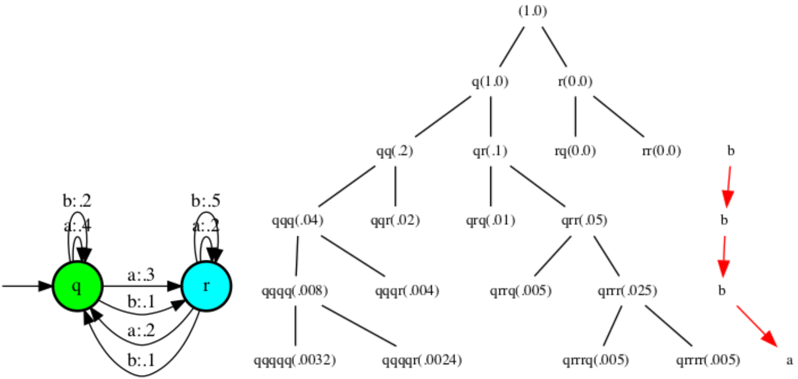
$$backpointer[q_F,T] \leftarrow \operatorname*{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step

return the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

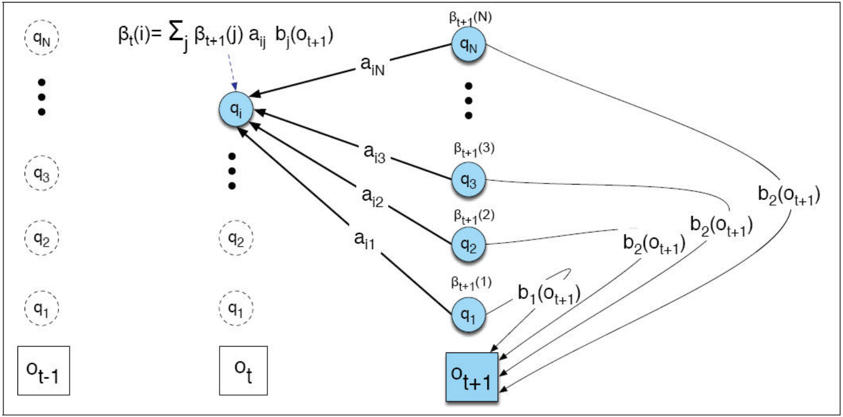
Viterbi with Backtrace



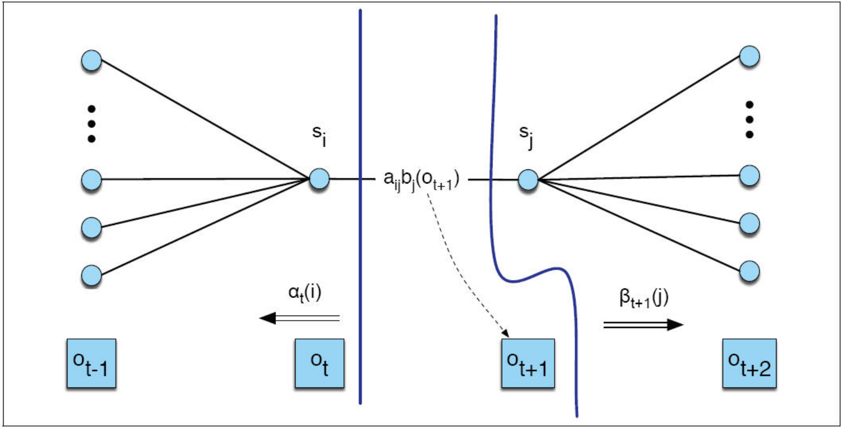
An example


b:.2
a:.4
b:.5
a:.2
q
r
a:.3
b:.1
a:.2
b:.1
(1.0)
q(1.0)
r(0.0)
qq(.2)
qr(.1)
rq(0.0)
rr(0.0)
b
qqq(.04)
qqr(.02)
qrq(.01)
qrr(.05)
b
qqqq(.008)
qqqr(.004)
qrrq(.005)
qrrr(.025)
b
qqqqq(.0032)
qqqqr(.0024)
qrrrq(.005)
qrrrr(.005)
a

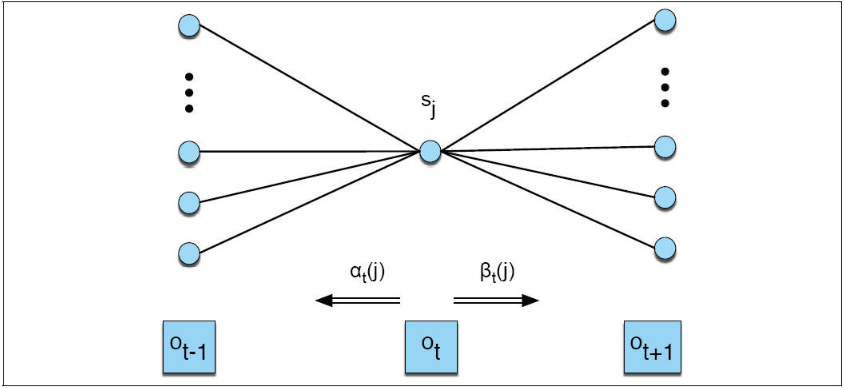
Backward Prob ($\beta(q_i)$)



Forward prob: one component



Combining α and β Probs



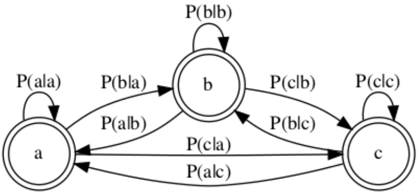
HMMs extend Markov Models

Markov Model: Sequential probability model depending on a limited history

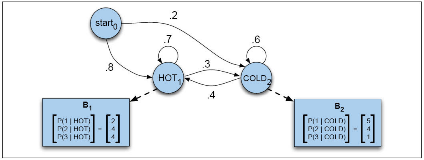
Chain States correspond directly to observations. State transitions depend only on state history

HMM States encode "hidden information" on which observations and state transitions depend. Most important use: Recover most likely "hidden" state sequence to produce a sequence of observations.

Chain



HMM



Most important HMM algorithms

Algorithm	Returns
Forward	Probability of an observation sequence
Viterbi	Most probable sequence of hidden states given an observation sequence
Forward-Backward	Learn an HMM model from a training set of observation sequences