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Matrix Differentiation - the following are some useful results.

(i) (Inner-product) Let a, b , and θ are vectors, then

$$\frac{\partial(a'b)}{\partial \theta} = \left(\frac{\partial a'}{\partial \theta} \right) b + \left(\frac{\partial b'}{\partial \theta} \right) a$$

(ii) (Quadratic form) Let x is a vector and A is a symmetric matrix, then

$$\frac{\partial}{\partial x} x' A x = 2 A x$$

(iii) (Inverse) Let the matrix A depends on a vector θ and is nonsingular, then, for any component θ_c of θ ,

$$\frac{\partial A^{-1}}{\partial \theta_c} = - A^{-1} \left(\frac{\partial A}{\partial \theta_c} \right) A^{-1}$$

(iv) (Log-determinant) Let the matrix A above is also positive definite, then, for any component θ_c of θ ,

$$\frac{\partial}{\partial \theta_c} \log(|A|) = \text{trace} \left(A^{-1} \frac{\partial A}{\partial \theta_c} \right)$$