

STAT 700, Fall 2011  
Homework 8 Problems  
due Wed. November 9

2 Problems.

1. Return to Homework 7, **Dyestuff Data**: (Ref: Davies, 1960) The variation of the strength of (coloring powder) of a dyestuff from one manufacturing batch to another was studied. Strength was measured by dyeing a square of cloth with a standard concentration of dyestuff and visually comparing the result with a standard. The result was numerically scored as the percentage strength of the dyestuff. Large samples were taken from six batches and from each batch six subsamples were taken. The 36 subsamples were submitted to the laboratory in a random order for testing as described above. There are two sources of variability: batch-to-batch variability and measurement error.

We return to the one-way ANOVA model,

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$$

where  $\mu$  is the overall mean level,  $\alpha_i$  is the random effect of the  $i$ th batch and they are iid  $N(0, \sigma_\alpha^2)$  and  $\varepsilon_{ij}$  are iid  $N(0, \sigma^2)$ .

To get the data, source the file off the class web page:

```
> dye <- read.table('http://www.rohan.sdsu.edu/~babailey/stat700/dye.dat',  
header=T)
```

We will ignore the Subsample and make the Batch a factor by,

```
> dye$Batch <- as.factor(dye$Batch)
```

(a) We can use the R function `lme` to fit the random-effects model using REML (restricted maximum likelihood, the default). Your call should look like,

```
> fit <- lme(Strength~1, data=dye, random=~1 | Batch)
```

Use the summary function to get the REML estimates for the  $\sigma^2$  and  $\sigma_\alpha^2$ . How do these estimates compare with your estimates from Homework 7?

(b) The plot function will give you a diagnostic plot for your fit. From this plot, how well does the model fit the data?

(c) Test the hypothesis  $H_0 : \sigma_\alpha^2 = 0$  vs  $H_1 : \sigma_\alpha^2 \neq 0$ , using a LRT. What do you conclude?

2. over

2. Let observations  $Y_1, \dots, Y_n$  be described by the relationship,

$$Y_i = \theta X_i^2 + \varepsilon_i$$

where  $X_1, \dots, X_n$  are fixed constants and the  $\varepsilon_1, \dots, \varepsilon_n$  are iid  $N(0, \sigma^2)$ .

(a) Find the MLE of  $\theta$ ,  $\hat{\theta}$ .

(b) Find  $E(\hat{\theta})$  and  $Var(\hat{\theta})$ .

(c) For students that have taken STAT 670AB, find the best unbiased estimator of  $\theta$ .  
(Hint: CRLB).