

STAT 700
Homework 1 Problems
due Thursday September 11

Show all work.

1. Suppose we fit the model,

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (1)$$

where \mathbf{X} involves $r = 3$ independent variables. Assume that there are $n = 10$ observations and ε_i are independent $N(0, \sigma^2)$ random variables.

(a) Write model (1) in full matrix notation, indicating all individual elements and dimensions of matrices. You may assume there is an intercept.

(b) Give the form of the least squares estimator of $\boldsymbol{\beta}$, for the fit to model (1). Call it $\hat{\boldsymbol{\beta}}$. You may use the notation of model (1).

(c) Find $E(\hat{\boldsymbol{\beta}})$ for model (1). Is $\hat{\boldsymbol{\beta}}$ a biased or unbiased estimator of $\boldsymbol{\beta}$? Explain. Find the distribution of $\hat{\boldsymbol{\beta}}$.

Now assume that the true model involves an additional $s = 2$ independent variables contained in \mathbf{W} , so the true model is

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\boldsymbol{\gamma} + \boldsymbol{\varepsilon} \quad (2)$$

where $\boldsymbol{\gamma}$ is the vector of regression coefficients for the independent variables contained in \mathbf{W} .

(d) Write model (2) in full matrix notation, indicating all individual elements and dimensions of matrices.

(e) Under the true model (2), find $E(\hat{\boldsymbol{\beta}})$. In general, is $\hat{\boldsymbol{\beta}}$ an unbiased estimator of $\boldsymbol{\beta}$? Explain. Find the distribution of $\hat{\boldsymbol{\beta}}$.

2. Consider the linear model from class,

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}.$$

Assume that the ε_i are independent $N(0, \sigma^2)$ random variables or equivalently

$$\mathbf{Y} \sim N_n(\mathbf{X}\boldsymbol{\beta}, \sigma^2 \mathbf{I}_n).$$

Also, assume that $\mathbf{X}'\mathbf{X}$ is invertible.

The prediction of a future observation, $Y_0 = \mathbf{x}'_0\boldsymbol{\beta} + \varepsilon_0$ at a given vector of independent variables \mathbf{x}'_0 , is given by $\hat{Y}_0 = \mathbf{x}'_0\hat{\boldsymbol{\beta}}$. Find the distribution of \hat{Y}_0 .