

STAT 700
Homework 1 Problems
due Wednesday September 7

2 Problems. Show all work.

1. Consider the linear model from class,

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}.$$

Assume that the ε_i are independent $N(0, \sigma^2)$ random variables or equivalently

$$\mathbf{Y} \sim N_n(\mathbf{X}\boldsymbol{\beta}, \sigma^2 \mathbf{I}_n).$$

Also, assume that $\mathbf{X}'\mathbf{X}$ is invertible.

The prediction of a future observation, $Y_0 = \mathbf{x}'_0\boldsymbol{\beta} + \varepsilon_0$ at a given vector of independent variables \mathbf{x}'_0 , is given by $\hat{Y}_0 = \mathbf{x}'_0\hat{\boldsymbol{\beta}}$. Find the distribution of \hat{Y}_0 .

2. Let X_1, X_2, \dots, X_n be independent normal random variables with means μ_i and variances σ_i^2 . Let $Y = \sum_{i=1}^n \alpha_i X_i$ where α_i are constants. Use moment generating functions to show that Y is normally distributed and find its mean and variance.

Recall, the moment generating function (mgf) of a normal random variable X with mean μ and variance σ^2 is

$$M_X(t) = e^{\mu t + \sigma^2 t^2 / 2}.$$