1. Solve the inequality \( 7 - |x+2| > 2 \), use interval notation.

\[ 7 - |x+2| > 2 \implies 7 - 2 > |x+2| \]

\[ 5 > |x+2| \]

Either \( x+2 < 0 \) or \( x+2 > 0 \)

Case 1: \( x+2 < 0 \) \( \implies |x+2| = -(x+2) \) \( \implies 5 > -(x+2) \)

\( -5 < x+2 \)

\( -7 < x \)

Case 2: \( x+2 > 0 \) \( \implies |x+2| = x+2 \) \( \implies 5 > x+2 \)

\( 3 > x \)

Solution: \( [-7, 3] \)

2. Find the domain of the function \( f(x) = \sqrt{x^2 - 81} \), in interval notation.

We know that for real solutions to exist (not imaginary/complex), we must be taking the square root of a nonnegative number. Therefore, we must have that:

\[ x^2 - 81 \geq 0 \]

\[ x^2 \geq 81 \]

\[ x \geq \pm \sqrt{81} = \pm 9 \]

In I.N. \( (-\infty, -9] \cup [9, \infty) \)

we use the brackets when we have \( \leq \) or \( \geq \).
3) Find the difference quotient for \( f(x) = \frac{x+y}{x-a} \)

\[
\frac{f(x+h) - f(x)}{h} = \frac{x+h+y}{x+h-a} - \frac{x+y}{x-a} = \frac{(x+h+y)(x-a) - (x+y)(x+h-a)}{(x+h-a)(x-a)h}
\]

\[
= \frac{x^2 + xh + 4x - 9x - 9h - 36}{(x+h-a)(x-a)h} - \frac{x^2 + xh + 4x + 4h - 36}{(x+h-a)(x-a)h}
\]

\[
= \frac{x^2 + xh - 9x + 9h + 36 - x^2 + xh + 5x - 4h + 36}{(x+h-a)(x-a)h}
\]

\[
= \frac{-13h}{(x+h-a)(x-a)h} = \frac{-13}{(x+h-a)(x-a)}
\]

4) A small company manufacturing surfboards. The profit \( P \) of selling \( x \) boards is given by \( P(x) = 50000 + 120x - 0.6x^2 \). How many boards should be made to maximize profit?

\( P(x) = -0.6x^2 + 120x + 50000 \) is a parabola facing downwards. The maximum is found by taking the \( y \)-value of the vertex. The vertex occurs at:

\[
\frac{-b}{2a} = \frac{-120}{2(-0.6)} = 100
\]

Then \( x=100 \) boards will maximize the profit.
(6) Find the y-intercept, domain, range, and horizontal asymptote at

\[ y = 4 + 3^{-x} \]

\[ y = 4 + \frac{1}{3^x} = \frac{1}{3^x} + 4 \]

y-intercept: \((0, 5)\)  
\[ x = 0, \quad \frac{1}{3^0} + 4 = \frac{1}{1} + 4 = 5 \]

Domain: We can't take any real number in an exponent and there is no number \(x\) such that \(3^x = 0\), so our function is defined for all real numbers: \((-\infty, \infty)\)

Range: \(\frac{1}{3^x} > 0\) for every \(x\) in our domain. As \(x \to \infty\), \(\frac{1}{3^x}\) becomes very small. This means we can make \(\frac{1}{3^x}\) as close to zero as we want, but we can never actually get to zero. If \(x\) is negative, then \(\frac{1}{3^x}\) will no longer be a fraction, and will grow unbounded. So range: \((4, \infty)\)

HA: From the range, HA: \(y = 4\)
6. Find fog and gof, with each domain.

\[ f(x) = |x - 7|, \quad g(x) = \frac{1}{5x} \]

Remember, \[ |x - 7| = |7 - x| \] Why? \[ |x - 7| = |(x - x + 7)| = |1||x + 7| = |1| |x + 7| = |x + 7| = |7 - x| \]

\[ f \circ g = f(g(x)) = 7 - \frac{1}{5x} = \frac{35x - 1}{5x} \quad \Rightarrow \quad \text{Domain} \ (-\infty, 0) \cup (0, \infty) \]

\[ g \circ f = \frac{1}{5|x - 7|} \quad \Rightarrow \quad \text{Domain} \ (-\infty, 7) \cup (7, \infty) \]

7. If is one-to-one, find its inverse and the domain/range of \( f \) and \( f^{-1} \).

\[ f = \frac{x}{\sqrt{x + 4}} \]

\[ y = \frac{x}{\sqrt{x + 4}} \quad \Rightarrow \quad x = \frac{y}{\sqrt{y + 4}} \]

Then, we solve for \( y \) to find \( f^{-1}(x) \).

\[ x = \frac{y}{\sqrt{y + 4}} \quad \Rightarrow \quad x\sqrt{y + 4} = y \quad \Rightarrow \quad \sqrt{y + 4} = \frac{y}{x} \quad \Rightarrow \quad y + 4 = \left(\frac{y}{x}\right)^2 = \frac{y^2}{x^2} \]

\[ x^2(y + 4) = y^2 \quad \Rightarrow \quad x^2y + 4x^2 = y^2 \quad \Rightarrow \quad 0 = y^2 - x^2y - 4x^2. \]

Now use the quadratic formula with respect to \( y \).

\[ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ y = \frac{-(-x^2) \pm \sqrt{(-x^2)^2 - 4(1)(-4x^2)}}{2(1)} = \frac{x^2 \pm \sqrt{x^4 + 16x^2}}{2} \]

\[ x^2 = \frac{x^2 \pm \sqrt{x^4 + 16x^2}}{2} \]

\[ x^2 = \sqrt{x^4 + 16x^2} \]

Which one? Try plugging in values:

\[ f(x) = y \quad \text{then} \quad f^{-1}(y) = x \]

\[ f(12) = \frac{12}{\sqrt{2} + 4} = \frac{12}{5} = 3 \]

So, \( f^{-1}(3) = 12 \), \( f^{-1}(9) = \frac{3 \pm \sqrt{9^2 + 16}}{2} = \frac{3 \pm \sqrt{85}}{2} \)

So, \( f^{-1}(x) = \frac{x^2 + x\sqrt{x^2 + 16}}{2} \)

\[ \text{Domain: } (-\infty, 0) \nLeft \rightarrow \nRight \rightarrow \n\text{Range: } (0, \infty) \]

\[ f \text{ Domain: } (11, 0) \nLeft \rightarrow \n\text{Range: } (0, 11) \]
(1) Given a zero of \( p(x) \), determine all other zeros.

\[ p(x) = x^4 - 15x^2 + 77x - 140 \]

We know \( 2 + i \) is a zero because imaginary zeros come in conjugate pairs.

So \( (x - (2+i))(x - (2-1)) = (x - 2 - i)(x - 2 + i) = (x-2)^2 - i^2 = x^2 - 4x + 4 + 1 \)

Now divide:

\[ x^2 - 4x + 5 \]

\[ \frac{x^4 - 15x^2 + 77x - 140}{x^2 - 11x + 28} \]

\[ x^2 - 4x + 5 \]

\[ - \frac{x^4 - 4x^3 + 5x^2}{0 - 11x^2 + 167x + 140} \]

\[ - \frac{-11x^2 + 144x - 55x}{0 + 2x^2 - 112x + 140} \]

\[ - \frac{2x^2 - 112x + 140}{0} \]

Now factor: \( x^2 - 11x + 28 = (x - 7)(x - 4) \) \( \Rightarrow x = 7, 4 \)

and \( p(x) = (x - 7)(x - 4)(x - (2+i))(x - (2-1)) \)

(2) Graph: \( F(x) = \frac{-3x^2}{x^2 + 4} \)

Note: Horizontal Asymptote = \( \frac{-3}{4} \) since exponents are equal.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
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<tr>
<td>-3</td>
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<tr>
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<td>-13/8</td>
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<tr>
<td>3</td>
<td>-27/13</td>
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</tbody>
</table>
15. Solve the exponential exactly for $x$.

\[ \frac{x^2 - 8}{\ln 2} = 2^x \]

\[ \iff \quad x^2 - 8 = (\ln 2)^x \]

\[ \iff \quad (x^2 - 8) \log_{10} 2 = 2x \log_{10} 2 \]

\[ x^2 - 2x - 8 = 0 \]

\[ (x - 4)(x + 2) = 0 \]

\[ x = 4, -2 \]

16. Graph $y = \log_2 (x + 5) - 2$.

\[ \log_2 x \implies \log_2 (x + 5) \implies \log_2 (x + 5) - 2 \]

17. Find amplitude, period, phase shift.

\[ y = 5 \sin (\pi x - 3) \]

Amplitude $= 5$

Period $= \frac{2\pi}{\pi} = 2$

Phase Shift $= \frac{3}{\pi}$

\[ 5 \sin (\pi x - 3) = 5 \sin \left( \pi \left( x - \frac{3}{\pi} \right) \right) \]
22) \[ \text{Find } \cos \left( \frac{4\pi}{3} \right) \text{ in Q3.} \] 

\[ \text{so } \cos \frac{4\pi}{3} = -\frac{1}{2} \]

23) \[ \text{Solve } y \cos \left( \frac{1}{2} \theta \right) = -2\sqrt{2} \text{ on } 0 \leq \theta < 2\pi \]

\[ \cos \left( \frac{1}{2} \theta \right) = \frac{-2\sqrt{2}}{y} = -\frac{\sqrt{2}}{2} \text{ in QII} \]

\[ \frac{\theta}{2} = \cos^{-1} \left( \frac{-\sqrt{2}}{2} \right) = \frac{3\pi}{4} \]

\[ \Rightarrow \theta = 2 \left( \frac{3\pi}{4} \right) = \frac{3\pi}{2} \]

24) \[ \text{Find } \Theta \text{ such that } \cos \Theta = -1 \text{ in QII.} \]

\[ \cos \Theta = -1 \Rightarrow \cos \Theta = -\sin \Theta \]

\[ \cos \Theta > 0 \text{ if } \Theta \text{ in Q1 or Q4} \]

\[ \sin \Theta < 0 \text{ if } \Theta \text{ in Q3 or Q4} \]

\[ \text{so } \Theta = \frac{3\pi}{4} \text{ since } \frac{\cos \Theta}{\sin \Theta} = -\frac{\sqrt{2}}{2} = -1 \]
25. Simplify \( \sec^2 x - \tan^2 x \)

\[ \cos^2 x + \sin^2 x = 1 \implies 1 + \tan^2 x = \sec^2 x \]

So \( \sec^2 x - \tan^2 x = (1 + \tan^2 x) - \tan^2 x = 1 \)

26. Write as single trigonometric function

\[ \sin (6x) \sin (11x) + \cos (6x) \cos (11x) \]

\[ \cos A \cos B + \sin A \sin B = \cos (A-B) \]

\( \implies \sin 6x \sin 11x + \cos 6x \cos 11x = \cos (6x-11x) = \cos (-5x) = \cos (5x) \)

28. Simplify \( \frac{1 - \cos 135}{\sin 135} \)

\[ \tan \left( \frac{\theta}{2} \right) = \frac{1 - \cos \theta}{\sin \theta} \]

\[ = \tan \left( \frac{135}{2} \right) = \tan \left( 67.5 \right) \]

29. Write as product: \( \sin (0.6x) + \sin (0.8x) \)

\[ \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \]

\( \implies \sin (0.6x) + \sin (0.8x) = 2 \sin (0.7x) \cos (0.1x) \)

30. Evaluate \( \sin \left( \cos^{-1} \left( \frac{7}{8} \right) \right) \)

\[ \cos \theta = \frac{7}{8} \]

\[ \sin \left( \cos^{-1} \left( \frac{7}{8} \right) \right) = \frac{x}{8} = \frac{\sqrt{15}}{8} \]

\[ 8^2 = x^2 + 7^2 \]

\[ 8^2 - 7^2 = x^2 \]

\[ \sqrt{64 - 49} = x \]

\[ \sqrt{15} = x \]