M141 Final Exam - Saturday December 8, 2012 - FORM B

Name (PRINT CLEARLY) Solutions

Put your answer in the appropriate space(s), or select the appropriate multiple choice by circling your answer. Please show your student ID upon request. Please remove all hats (baseball, etc.). No calculators, notes, books, ipods, cell phones etc.

1. Find the domain of the function \( F(x) = \sqrt{x^2 - 81} \). Express the domain in interval notation.

- \((-\infty, -9) \cup (9, \infty)\)
- \((-9, 9)\)
- \((-\infty, \infty)\)
- \((-\infty, -9] \cup [9, \infty)\)
- \([-9, 9]\)

2. Solve the inequality \( 6 - |x+5| > 5 \) and express the solution in interval notation.

- \((-6, -4)\)
- \((-\infty, \infty)\)
- \([-6, -4]\)
- \((-\infty, -6) \cup (-4, \infty)\)
- \((-\infty, -6] \cup [-4, \infty)\)
3. Given the functions \( f \) and \( g \), find \( f \circ g \), \( g \circ f \), and the domains of each.

\( f(x) = |x - 10|, \ g(x) = \frac{1}{5x} \).

The functions and their domains are:

\[ f \circ g = \frac{50x - 1}{5x}, \ g \circ f = \frac{1}{5|x - 10|} \]

Domain of \( f \circ g \): \((-\infty, \infty)\)

Domain of \( g \circ f \): \((-\infty, \infty)\)

\[ f \circ g = \frac{50x - 1}{5x}, \ g \circ f = \frac{1}{5|x - 10|} \]

Domain of \( f \circ g \): \((-\infty, 0) \cup (0, \infty)\)

Domain of \( g \circ f \): \((-\infty, 10) \cup (10, \infty)\)

\[ f \circ g = \frac{1}{50|x - 10|}, \ g \circ f = \frac{50x - 1}{5x} \]

Domain of \( f \circ g \): \((-\infty, 0) \cup (0, \infty)\)

Domain of \( g \circ f \): \((-\infty, 0) \cup (0, \infty)\)

\[ f \circ g = \frac{1}{5|x - 10|}, \ g \circ f = \frac{50x - 1}{5x} \]

Domain of \( f \circ g \): \((-\infty, 0) \cup (0, \infty)\)

Domain of \( g \circ f \): \((-\infty, 0) \cup (0, 10) \cup (10, \infty)\)
4. Find the difference quotient \( \frac{f(x+h)-f(x)}{h} \) for the function \( f(x) = \frac{x+3}{x-9} \).

\[
\begin{align*}
\frac{f(x+h) - f(x)}{h} &= \\
&= \frac{12}{(x+9+h)(x-9)} \\
&= \frac{3}{(x-9+h)(x-9)} \\
&= \frac{12}{(x-9+h)(x-9)} \\
&= \frac{3}{(x-9+h)(x-9)}
\end{align*}
\]

5. **Profit.** A small company in Virginia Beach manufactures handcrafted surfboards. The profit of selling \( x \) boards is given by \( P(x) = 30,000 + 120x - 0.6x^2 \). How many boards should be made to maximize the profit?

\[
\begin{array}{c}
\circ & 200 \\
\circ & 120 \\
\circ & 150 \\
\bullet & 100 \\
\circ & 110
\end{array}
\]

6. Graph on scratch paper the exponential function \( y = 3 + 3^{-x} \) using transformations. Do not hand in (not graded). From this graph state the \( y \)-intercept, the domain, the range, and the horizontal asymptote.

- **\( y \)-intercept:** \( (0, 4) \) (or 4)
- **Domain:** \( (-\infty, \infty) \)
- **Range:** \( (3, \infty) \)
- **Horizontal asymptote:** \( y = 3 \)
7. Given a zero of the polynomial, determine all other zeros (real and complex) and write the polynomial in terms of a product of linear factors. 
\[ P(x) = x^4 - 11x^3 + 43x^2 - 75x + 50; \text{ Zero: 2} - i. \]

Ø zeros: \(- 2, 5, 2 + i, 2 - i\)
\[ P(x) = (x + 2)(x - 5)(x - (2 + i))(x - (2 - i)) \]

Ø zeros: \(2, 5, 2 + i, 2 - i\)
\[ P(x) = (x - 2)(x - 5)(x - (2 + i))(x - (2 - i)) \]

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Ø zeros: \(- 2, 5, 2 + i, 2 - i\)
\[ P(x) = (x - 2)(x + 5)(x - (2 + i))(x - (2 - i)) \]
8. The function $f$ is one-to-one. Find its inverse and determine the domain and range of both $f$ and $f^{-1}$. \[ f(x) = \frac{x}{\sqrt{x+1}}. \]

<table>
<thead>
<tr>
<th>$f^{-1}$</th>
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<tbody>
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9. Rewrite the quadratic equation in standard form by completing the square. \[ f(x) = -\frac{1}{3}x^2 + 6x + 4. \]

- $\frac{1}{3}(x - 9)^2 + 31$
- $-\frac{1}{3}(x - 9)^2 + 31$
- $-\frac{1}{3}(x - 9)^2 + 23$
- $-\frac{1}{3}(x - 9)^2 - 23$
- $-\frac{1}{3}(x - 9)^2 - 31$
10. Given \( x = 4 \) and \( x = -2 \) are zeros of the polynomial
\[ P(x) = x^4 - 14x^3 + 18x^2 + 92x - 16 \], determine all other zeros and write the polynomial in terms of a product of linear and/or irreducible factors.

- Zeros: \(-2, 4\)
  \[ P(x) = (x + 2)^2 (x - 4)^2 \]

- Zeros: \(-2, 4\)
  \[ P(x) = (x^2 - 12x + 2)(x - 2)(x - 4) \]

- Zeros: \(-2, 4\)
  \[ P(x) = (x^2 - 12x + 2)(x + 2)(x - 4) \]

- Zeros: \(-2, 4\)
  \[ P(x) = (x^2 - 12x + 16)(x + 2)(x - 4) \]

11. Use synthetic division to divide the polynomial by the linear factor
\( (4x^2 + x + 7) \div (x - 2) \). Indicate the quotient, \( Q(x) \), and the remainder, \( r(x) \), in the following equation:
\[ \frac{P(x)}{d(x)} = Q(x) + \frac{r(x)}{d(x)} \], where \( P(x) = 4x^2 + x + 12 \) and \( d(x) = x - 2 \).

- \( Q(x) = 4x - 7 \), \( r(x) = -11 \)
- \( Q(x) = 4x + 9 \), \( r(x) = 7 \)
- \( Q(x) = 4x + 9 \), \( r(x) = -25 \)
- \( Q(x) = 4x + 9 \), \( r(x) = 25 \)
- \( Q(x) = 4x - 7 \), \( r(x) = 11 \)
12. Match the function $f(x) = \frac{-3x^2}{x^2 - 4}$ to the correct graph.
13. Convert the exponential equation $3 = \sqrt[4]{81}$ to its equivalent logarithmic form.

- $\log_{81} 3 = \frac{1}{4}$
- $\log_{81} 4 = \frac{1}{3}$
- $\log_{81} \frac{1}{3} = 4$
- $\log_{4} \frac{1}{3} = 81$
- $\log_{4} 81 = \frac{1}{3}$

14. Apply the properties of logarithms to simplify $2e^{-3\ln(x)}$.

- $\frac{2}{x^3}$
- $2x^3$
- $-2x^3$
- $-\frac{2}{x^3}$
- $-3x^2$

15. Solve the exponential equation exactly for $x$. Enter your answers in increasing order. If the answer is doubled, enter it twice. $5^{x^2-3} = 25^x$.

- $x = \frac{1}{2} - \frac{1}{2} \sqrt{13}, x = \frac{1}{2} + \frac{1}{2} \sqrt{13}$
- $x = -1, x = 3$
- $x = -3, x = 1$
- $x = -1, x = -1$
- $x = -\sqrt{5}, x = \sqrt{5}$
16. Choose the graph that matches the function $y = \log_2(x + 4) - 2$. 
17. Use the following triangle to find $\sec(\theta)$. Rationalize any denominators containing radicals that you encounter in the answer.

- $\frac{\sqrt{17}}{3}$
- $\frac{1}{\sqrt{17}}$
- 4
- $\frac{1}{4}$
- $\sqrt{17}$

18. State the amplitude, period, and phase shift (including direction) of the function. $y = 3\sin(\pi x - 5)$. Enter the exact answers. Enter a positive number for the phase shift and indicate whether it is a shift to the right or left.

The amplitude is 3
The period is 2
The phase shift is $\frac{5}{\pi}$ units to the left (circle one).
19. Write the expression as a sum or difference of logarithms. \( \log \sqrt[2]{\frac{x^2 + 4x - 12}{x^2 - 7x + 10}} \).

- \( \frac{1}{2} \log(x - 6) + \frac{1}{2} \log(x + 2) - \frac{1}{2} \log(x - 5) - \frac{1}{2} \log(x - 2) \)
- \( \frac{1}{2} \log(x + 6) - \frac{1}{2} \log(x - 2) \)
- \( \frac{1}{2} \log(x - 6) - \frac{1}{2} \log(x + 5) \)
- \( \frac{1}{2} \log(x - 6) + \frac{1}{2} \log(x + 5) - \frac{1}{2} \log(x - 5) - \frac{1}{2} \log(x - 2) \)
- **\( \frac{1}{2} \log(x + 6) - \frac{1}{2} \log(x - 5) \)**

20. Solve the logarithmic equation exactly. Enter your answer in fraction form.
\( \log_2 (3x - 4) = -3 \).

- \( \frac{5}{11} \)
- \( -\frac{11}{8} \)
- 4
- **\( \frac{11}{8} \)**
- \( \frac{8}{11} \)

21. Find all values of \( \theta \), where \( 0^\circ \leq \theta \leq 360^\circ \), when \( \sin(\theta) = -\frac{\sqrt{3}}{2} \). Enter your answers in increasing order.

- \( \theta = 210^\circ, \theta = 300^\circ \)
- \( \theta = 210^\circ, \theta = 150^\circ \)
- **\( \theta = 240^\circ, \theta = 300^\circ \)**
- \( \theta = 150^\circ, \theta = 240^\circ \)
- \( \theta = 60^\circ, \theta = 120^\circ \)
22. Evaluate the given expression exactly as a fraction.

\[ \sin \left[ \cos^{-1} \left( \frac{3}{4} \right) \right] = \]

- \( \frac{\sqrt{7}}{3} \)
- \( \frac{3}{4} \)
- \( \frac{7}{3} \)
- \( \frac{\sqrt{7}}{4} \)
- \( -\frac{\sqrt{7}}{3} \)

23. Simplify the expression using half-angle identities. Do not evaluate.

\[ \frac{1 - \cos(225^\circ)}{\sin(225^\circ)}. \]

- \( \tan \ 112.5^\circ \)
- \( \tan \ 225^\circ \)
- \( \cos \ 112.5^\circ \)
- \( \sin \ 112.5^\circ \)
- \( \cot \ 225^\circ \)

24. Write the expression \( \sin(0.4x) + \sin(0.6x) \) as a product of sines and/or cosines.

- \( 2\cos(0.1x)\cos(0.5x) \)
- \( 2\sin(0.1x)\cos(0.5x) \)
- \( 2\sin(0.5x)\cos(0.1x) \)
- \( -2\sin(0.5x)\cos(0.1x) \)
- \( 2\sin(0.5x)\sin(0.1x) \)
25. Find the exact value of $\cos\left(\frac{5}{3}\pi\right)$ using the unit circle.

- $\frac{1}{2}$
- $\frac{\sqrt{3}}{2}$
- $\frac{\sqrt{3}}{2}$
- $\frac{-\sqrt{3}}{2}$
- $\frac{-\sqrt{2}}{2}$

26. Solve $2\cos\left(\frac{\theta}{2}\right) = -\sqrt{3}$ exactly on $0 \leq \theta < 2\pi$.

- $\frac{\pi}{3}$
- $\frac{\pi}{6}$
- $\frac{\pi}{4}$
- $\frac{\pi}{3}$
- $\frac{\pi}{2}$
- $\frac{1}{6}\pi$

Do not write in this square: [ ]
27. Simplify the following trigonometric expression. \( \sec^2(x) - \tan^2(-x) \).

- \( 2\sec^2 x + 1 \)
- \( -1 \)
- 1
- \( \tan x \)
- \( 2\tan^2 x + 1 \)

28. Use the unit circle (see question 25) to find all of the exact values of \( \theta \) (in radians) that make the equation true in the indicated interval. \( \cot(\theta) = -1 \) \( (0 \leq \theta \leq 2\pi) \). Enter your answers in increasing order.

- \( \theta = \frac{1}{4}\pi, \theta = \frac{3}{4}\pi \)
- \( \theta = \frac{3}{4}\pi, \theta = \frac{7}{4}\pi \)
- \( \theta = \frac{3}{4}\pi, \theta = \frac{5}{4}\pi \)
- \( \theta = \frac{5}{4}\pi, \theta = \frac{7}{4}\pi \)
- \( \theta = \frac{1}{4}\pi, \theta = \frac{7}{4}\pi \)

29. Write the expression as a single trigonometric function.

\( \sin(7x)\sin(8x) + \cos(7x)\cos(8x) \).

- \( \cos(x) \)
- \( -\cos(x) \)
- \( \cos(15x) \)
- \( \sin(15x) \)
- \( -\sin(x) \)
30. Match the function \( y = -\tan(2x) \) with the appropriate graph.

Do not write in this square: