

April 28, 2005

Math 121a

Name Key

Instructor (Chávez-Ross or Mahaffy - circle)

Lab Section (Th8 Th10 F8 F10 - circle)

Give all answers to at least 4 significant figures.

1. Over the summer a group of ecology graduate students collects data from a number of water samples with copepods. Below is a table showing the average data that they collected in different weeks.

Week	Population/cc
0	35.6
1	38.7
2	42.8
4	36.2
5	31.2
7	18.4
9	11.9
10	15.4
11	22.7
13	27.5
14	26.2
15	16.5

a. Fit the best fourth order polynomial through the data of the form

$$P(t) = a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0.$$

Be sure that all of your coefficients have 4 significant figures. Write your best fitting model to the data.

$a_4 = \underline{-0.01787}$ $a_3 = \underline{0.5596}$ $a_2 = \underline{-5.3911}$

$a_1 = \underline{14.266}$ $a_0 = \underline{32.954}$

b. Differentiate $P(t)$ and give the formula for the derivative.

$P'(t) = \underline{-0.07148t^3 + 1.6788t^2 - 10.782t + 14.266}$

c. Find all relative minima and maxima during the time of the survey, $t \in [0, 15]$. Be sure to give both the values of t and P (to 4 significant figures).

$t_{1c} = \underline{1.7789}$ $P(t_{1c}) = \underline{44.246}$ Max or Min Max

$t_{2c} = \underline{8.4918}$ $P(t_{2c}) = \underline{15.127}$ Max or Min Min

$t_{3c} = \underline{13.2156}$ $P(t_{3c}) = \underline{26.523}$ Max or Min Max

2. An experimental drug is being tested to see the response of certain hormones in the body. Below is a table of the readings of the drug in the blood as time passes after an injection of the drug at $t = 0$ hours.

Hour	Drug ($\mu\text{g}/\text{dl}$)
0	7.0
1	6.6
2	6.3
4	5.6
6	5.0
10	4.1
15	3.0
20	2.3

a. The drug is metabolized in the liver and eliminated through the urine. It is assumed that the decay of this drug is exponential, so fits the model

$$D(t) = A e^{-kt}.$$

Use Excel's Trendline with an Exponential fit to the data. Give the best values of A and k that you find (to 4 significant figures). Differentiate $D(t)$ to find the rate of loss of the drug.

$A = \underline{7.0138} \quad k = \underline{0.05577}$

$D'(t) = \underline{-0.3912 e^{-0.05577t}}$

b. The drug stimulates a hormone response in the body. Over the next 20 hours the blood is measured to determine the response of the body to this new experimental drug. Below are the data for the level of hormone in the blood.

Hour	Hormone (ng/dl)
0	0
1	17.2
2	31.4
4	52.1
6	64.9
10	74.6
15	70.3
20	58.8

The researchers use a Ricker's model to simulate the hormone release from a specialized group of cells (a type of population model for the response). Their best fit to the data is the model

$$R(t) = 18.9t e^{-0.093t}.$$

Find the sum of square errors between this model and the data.

Sum of Square Errors = 0.00533

Give the percent error at $t = 10$ and $t = 20$ hours. For percent error use

$$\text{Error} = 100 \frac{(\text{Model} - \text{Data})}{\text{Data}}.$$

3 Percent Error at $t = 10$ is -0.03934 Percent Error at $t = 20$ is 0.07526

Find the derivative of this model ($R'(t)$).

4 $R'(t) = \underline{(18.9 - 1.7577t) e^{-0.043t}}$

Use the techniques from class to find the time that this model predicts a maximum concentration of the hormone and what that concentration is. (Give your answers to 4 significant figures.)

6 $t_{\max} = \underline{10.7527}$ $R(t_{\max}) = \underline{74.7626}$