



Facilitator's Guide

***RANKING DATA
TO MAKE DECISIONS***

The Case of the Sneaker Purchase

A Case Study

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Introduction

Case studies have a long and rich history of use in engineering, law, and business. The use of case studies in education is a more recent phenomenon and would appear to be a practice that holds some promise for bringing new practitioners into the profession of teaching. This particular case study was designed to help preservice teachers understand the complexity of the classroom, become better observers of classroom interactions, and learn to reflect on their own developing practice. This case study can provide a common context within which interesting aspects of a mathematics lesson can be discussed. In doing so, this case study can help highlight some of the incidents and issues that are critical in mathematics education.

As a facilitator of this case study, you will need to become familiar with the lesson plans, the reflections of the teacher, and the activity of the students in this seventh-grade class. The easiest way to do this, of course, is to read, watch, and listen to the Video Overview (background on the teacher, the school, the students, and the lesson). After reading the Lesson Plans (written plans of the teacher for this lesson in data analysis), you should listen to the teacher's reflections, where she discusses her plans and intentions for this lesson.

The classroom video contains 30 minutes of edited video. It begins with the teacher introducing the "Sneaker" problem. This is followed by the work of three groups of students and then the posing of the crucial second part of the problem: how to combine the ranked lists of data that each of the groups generated. This is followed by a video of the same three groups of students working on this problem and then the whole-class discussion of different groups' solutions. The last section of the classroom video is from the first ten minutes of the next day, where the teacher recaps and brings some closure to mathematical questions left from the previous day.

The reflections video contains two sections. The first section includes the teacher's reflections on her planning process. For ease of viewing, these reflections have been divided into ten different topics as noted on the timeline. A legend describing each section is located in the bookmark section of this video.

Legend

- | | |
|---|--|
| 1. Planning the sequence of mathematical concepts | 6. Determining optimal group size |
| 2. Background activities | 7. Making decisions in action |
| 3. Anticipating students' mathematical solutions | 8. Supporting the development of social norms |
| 4. Engaging all the students in the problem | 9. Balancing management issues and mathematical issues |
| 5. Planning the rhythm of the class | 10. Monitoring small groups and structuring whole class discussion |
-

The final video section features the teacher's reflections on the lesson itself. She describes several key points, including the unanticipated issues that arose, and how she dealt with them.

Key features of this multimedia case study include:

- (a) full text of the teacher's lesson plan;
- (b) teacher's reflections prior to and following the class lesson;
- (c) seating chart that allows the user to identify each student by face and seat location, and also to view each group's initial work;
- (d) issues matrix with links to the video;
- (e) transcripts that scroll along with the video;
- (f) notebook in which the preservice teacher can make annotations while investigating this case study;
- (g) bookmarks which can be customized to flag incidents of interest to the instructor; and
- (h) search feature for finding text in the transcript.

Facilitating the Case Study

The complexities of any classroom provide the opportunity for numerous starting points and navigational paths through the interactions and activities of the participants. One way through this case study is to begin with the *Mathematics of the Case*. This can be followed by reading, watching, and listening to the introductory overview, the lesson plans, and the teacher's anticipations of the lesson. This provides an opportunity to begin your case study investigation with a brief discussion of the *Background Facts of the Case*.

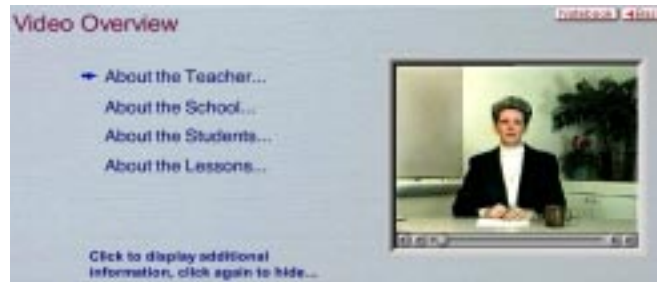
This can be followed by viewing the first half of the lesson, up to where the students complete their work in small groups to aggregate the data. This provides a natural break in the lesson where you might want the preservice teachers to discuss how they would orchestrate the whole-class discussion that follows. This would also be a point at which you could use the discussion *Questions Raised by the Case* to raise salient issues or to focus issues that arise for your students.

Watching the whole-class discussion in the lesson and the teacher's reflections should provide numerous opportunities for preservice teachers to discuss pedagogical strategies, student thinking and mathematical content. This is a useful juncture at which to ask the preservice teachers how they would plan for the next day's lesson and to explore the mathematical content of this lesson. The last segment of the case study shows the beginning of the second day's lesson and how the teacher approached the dilemmas and open issues from the first day's lesson.

Resources

The resources in this multimedia case study include:

Video Overview



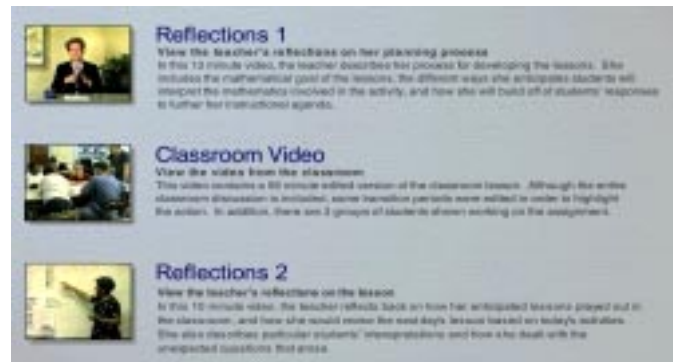
- This feature gives viewers an overview of the teacher's background, the school setting, the students, and a general description of the lesson.

Lesson Plans



- This feature enables viewers to read, copy, and print the full text of the lesson plan.

Video Selection

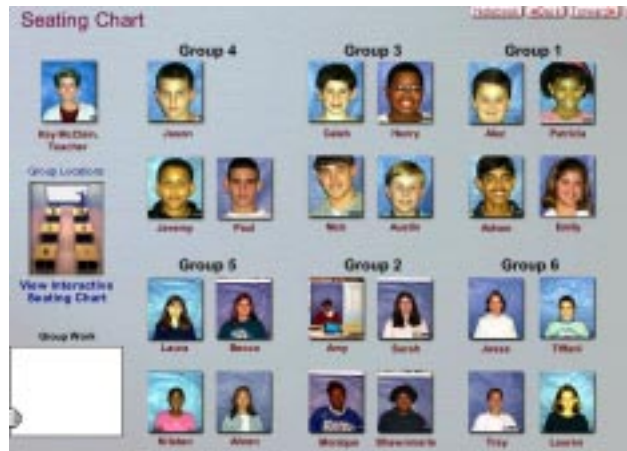


- This feature enables viewers to select a video segment. The three included in this case are: the teacher's reflections on her lesson plans prior to the lesson, the lesson itself, and the teacher's reflections on the class after the lesson.

Case Traversal Tools

To assist you and your preservice teachers in using the resources in this multimedia case study, we designed the following tools:

Seating Chart



- The seating chart feature was designed to help viewers identify students by name.

Each student is organized by group. The 3-dimensional seating chart icon helps to position each group within the physical layout of the classroom.

Issues Matrix

	Reflections 1	Lesson Plan	Classroom Reflections Lesson 2
PLANNING			
Goal of lesson	X	X	X
Instructional sequence	X		X
Lesson Organization	X	X	X
Anticipating Student Understanding	X		
FACILITATING			
Managing Transitions	X		
Facilitating Group Work	X X		X X
Building on Students thinking	X	X	X X
Classroom Norms	X	X	X X X X
ASSESSING			
Student Understanding	X	X	X X X X
Unanticipated Issues	X		X
CONTEXT OF PROBLEM	X		X X X X
MATHEMATICAL CONTENT		X	
Ranking	X		X
Frequency	X		X X
Sum			X X
Average	X		
Reverse sum			X
STUDENT GENERATED QUESTIONS			X X

- Table of hyperlinks to predefined issues with classroom video, teacher reflection video, and lesson plan text. Issues covered include:
 - Planning
 - Facilitating
 - Assessing students' understanding
 - Math content
 - Student generated questions

Bookmarks



- The bookmarking feature enables users to identify and annotate specific segments of video.

This feature can be used by instructors to show specific segments during lectures. Alternatively, it can be used by preservice teachers to study issues over the course of the class.

Notebook



- The notebook feature enables users to copy and paste text from the case into one central location, and to add notes for later use in presentations and papers. For example, preservice teachers working in a group can make annotations while investigating the case study, and export these notes to each group member for later review.

Transcript Search



- The search tool allows viewers to find any specified text within the transcript of the selected video. The search tool can be used to find one occurrence at a time, or it can generate an entire list of occurrences for later reference. This list, which is shown at left, contains hyperlinks that place the video and transcript at the starting point of the desired section.

Some Suggested Discussion Questions

The study questions included in this guide (and reflected in the issues matrix on the CD) are in four major categories: planning, facilitating, understanding student thinking and mathematical content and context. Using this case study for the first time, we suggest that you consider for written assignments for your preservice teachers the following questions:

Understanding Student Thinking #7 – to focus on the teacher’s understanding and building on student’s thinking.

Facilitating Questions #4, 5 or 6 – to focus on the teacher’s interactions with the small groups as they are working.

Mathematical Content and Context Question #11 – to elicit the preservice teachers’ understanding of the student strategies.

Planning Question #5 – to plan for ways to follow up on issues raised at the end of the first day’s lesson.

There are, of course, many ways to navigate the issues raised by this case study. Asking your preservice teachers to prepare written responses to some questions before your class meeting will hopefully bring out a diversity and richness of perspectives on teaching and learning.

The Mathematics of the Case

In using this case study with preservice teachers, we strongly recommend that the facilitator of the case begin by investigating the mathematics of the lesson with the preservice teachers. There are several important mathematical issues that this problem raises. Investigating the problem themselves as a learner of mathematics provides an opportunity for the preservice teachers to first understand the mathematics that is involved and then, later, through the case study to examine how middle-school students might approach this problem.

To investigate the mathematics, follow the teacher’s lesson plan with your own class. You will want to have at least 5 or 6 lists to combine into a single list. You can provide these lists for your students or have them generate them like the students in the 7th grade classroom. After the preservice teachers have worked the problem themselves, ask them to anticipate how 7th grade students might approach this task. Anticipating student responses is an integral and important aspect of planning any lesson. The teacher cannot build from students’ responses in a meaningful way if she has not anticipated how the students might complete a particular task. These anticipations have implications for not only orchestrating the whole-class discussion but for

planning subsequent lessons in the sequence. This also requires the teacher to think deeply about the mathematics that her students will be engaged in and how she might support her students' mathematical development.

The Background Facts of the Case

One way to begin the discussion of the case study is to ask the preservice teachers to list the facts of the case. This brief exercise enables the facilitator to help form a consensus about the context in which this case is set: the type of school, the experience of the teacher, the grade level of the students, the mathematical goals of the lesson and so on. This can be done very quickly, usually in about five minutes. This may also serve to focus on observational skills by noting what kinds of facts the preservice teachers attended to and what they (perhaps) ignored.

Questions Raised by the Case

The following questions are intended as suggestions for class discussion and/or for investigation by the preservice teacher using the case study as a resource. You may want to use some of these questions for written assignments to promote reflection, interpretation, observation and analysis by the preservice teacher. There is some overlap among the questions; they are not intended as a list of questions to be completed, but rather as starting points for discussion, analysis and reflection. The issues matrix provides links to the video that may be helpful. Refer to the seating chart if you have questions on student names.

Planning

1. One of the teacher's mathematical goals was to have this lesson build toward developing a strong notion in these students of the relationship between total and average. How did the teacher plan to go about meeting this goal? To what extent was this goal accomplished?
2. How does the teacher's ability to anticipate student understanding help her facilitate students' learning in class?
3. How does the teacher feel about obtaining closure and wrap up of a lesson?
4. What do you think about the teacher's level of planning and how it impacted her teaching in the classroom?
5. Near the end of the first day's lesson, Austin suggested that when ranks were averaged that this would result in the same ordered list as when the ranks were simply summed (as Alec's group had done). As the lesson continues, there is a confusion between division by eight and division by six to find the average.

Plan a “mini-lesson” that would use the next day to address these two issues. Compare your lesson to the video showing how the teacher addressed these issues at the beginning of day #2.

Facilitating

1. How did the teacher help students develop ways to give clear justifications and explanations? Can you think of additional methods to accomplish this goal?
2. How did the teacher encourage the students to listen to others and to ask questions of each other? Find several examples of student to student interactions during the whole-class discussion that reflect the norms for participation that the teacher is trying to support and develop.
3. The teacher clearly needed to decide whether or not to intervene with the groups. How did she go about making these decisions? What considerations did she need to take in account?
4. The teacher intervened with Group #1 (Adnan, Alec, Emily and Patricia) as they were devising their strategy for combining the lists. Describe the teacher’s interactions with this group. What was the teacher’s interpretation of what the students were doing? What do you think that the teacher intended to happen as a result of her interactions? What did happen? What other strategies could she have used in this situation? Why?
5. The teacher joined Group #2 (Monique, Sarah and Shawn-Marie) as they were discussing the lists. Describe the teacher’s interactions with this group. How do you think the teacher interpreted what the students were trying to do? What do you think that the teacher intended to happen as a result of her interactions? How did this differ from what the students actually did? What does this interaction suggest about the difficulty of facilitating small-group work?
6. As Group #3 (Austin, Caleb, Henry, and Nick) worked to prioritize their list of factors, they disagreed on which item should be listed first. The teacher overheard this conversation and offered an alternative way to approach the problem. Describe the method she offered. Use this episode as a basis for briefly describing your view on the teacher’s role in intervening with groups who are having difficulty proceeding with assigned tasks.
7. What issues did the students raise as the combined lists were posted on the board? How did the teacher use these issues to further the mathematical goals of the lesson? Why do you think the teacher asked: “So can you imagine what I am going to need your help with now?”
8. What attitudes do you believe the teacher holds towards the students in the class? Use episodes from the classroom and interviews to justify your response.

Understanding Student Thinking

1. What methods did the teacher use to assess student understanding? Can you think of additional assessment techniques?
2. What was the teacher's primary goal in conducting ongoing assessments of the students' understanding?
3. How did the teacher's assessments of the students' understanding effect her decisions about how to proceed with the lesson?
4. Listen to the conversation that Adnan and his group had with the teacher. How did they approach the task of combining the lists before the teacher started to work with them? How about after she left? Do you think the teacher understood what they were doing?
5. Tiffany questioned Becca's group on how they broke the tie between quality and brand. Was Becca's explanation understood by Tiffany? How can you tell? What would you (as the teacher) have said after her reply?
6. Find two or three instances of the teacher using questioning to elicit or clarify student thinking. How do these questions help the teacher as she moves forward in the lesson?
7. In order for the teacher to gauge the success of the lesson, it is important for her to be able to make judgments about the students' current understandings throughout the lesson. What strategies did the teacher use to achieve this goal?
8. Why do you think the teacher chose the order she did for the group presentations? How might the whole-class discussion have changed if the order of the presentations were reversed? How did the selection of student presentations support the teacher's mathematical agenda?

Mathematical Content and Context

1. What mathematics is involved in the assignment of combining six lists into one ranked list? What strategies might you anticipate students will devise?
2. What role did the context of purchasing sneakers play in this lesson? How did the teacher use the context and how did it influence the students' thinking?
3. The rationale for generating an aggregated list was to help the teacher in making a tennis shoes purchase. Some students were confused about the temporal issue, (i.e., what you consider first versus what you consider most important). See the discussion of Group #2 on this issue. Other students argued that some factors are so important as to exclude others. See Group #3's discussion of price. What other rationales or scenarios could the teacher have used for generating aggregated lists?

4. What information is lost when you create an aggregated list? What information is gained? When would you use an aggregated list like the ones the students created in this lesson?
5. To find the mean for “Comfort,” one group found the sum of the ranks ($2 + 5 + 1 + 1 + 4 + 2 = 15$) and then divided by 6 to obtain 2.5.
 - a) Interpret 2.5 in terms of the sneakers situation.
 - b) If the group had divided by 8 here to get 1.875, then what would 1.875 mean in terms of the situation?
6. One of the four foundational standards in the NCTM’s Curriculum Standards addresses the connections that students need to make both within mathematics and between mathematics and their world of experience. Describe a connection between the mathematics in this lesson and some other mathematical concept. Describe a connection between the mathematics in this lesson and another context in an every day experience of a middle-school student.
7. *Why* should the “average” method and the “sum” method produce the same results (assuming that the group correctly divides each sum by 6)? What would constitute a mathematical proof of that claim? Describe an acceptable mathematical argument you might expect from a 7th- or 8th-grader.

What if the group divided each sum by 8? Will these results and the “sum” method produce the same list? Why or why not?
8. Developing number sense is a critical goal for elementary and middle school mathematics. The calculation of the average is done by dividing the sum by a whole number. The mathematics here could be extended to consider other operations. Would the resulting lists be in the same order as the “sum” list if you divided by a fraction or by a negative number? Added a constant to the sum? Subtracted a constant from the sum? Or multiplied the sum by a constant? Why or why not?
9. Generate several other “answers” (i.e., aggregate data lists) by using a frequency or modal method. What are some other problems with the “frequency” method that the first group uses? What information does the modal method capture that the sum or average methods do not? What information is lost? Is one method “better” than another? Why or why not?
10. The last group to present their system for combining the individual lists into a single combined list for the class has an approach which is different from the previous groups, but for which the results are the same as Group #3 (Austin, Caleb, Henry, and Nick). Describe their system. What (if any) is the mathematical relationship between the sums which they found and the sums found by Group #1?

11. The six groups of students in this case study created the six lists shown in the table below.

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
1) price	1) brand	1) comfort	1) comfort	1) type	1) size
2) comfort	2) looks	2) quality	2) price	2) band	2) comfort
3) size	3) style	3) size	3) size	3) size	3) price
4) brand	4) price	4) price	4) quality	4) comfort	4) quality
5) quality	5) comfort	5) type	5) brand	5) price	5) type
6) looks	6) quality	6) style	6) style	6) quality	6) looks
7) style	7) size	7) looks	7) looks	7) style	7) style
8) type	8) type	8) brand	8) type	8) looks	8) brand

The groups then developed strategies to combine all of the lists into a single aggregated data list. The result of their work is shown in the table below.

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Comfort (15)	1. type	Comfort (2.5)	Comfort	1. Comfort	Comfort (39)
Size (20)	2. size	Price (3.2)	Price	2. Price	Price (35)
Price (22)	3. price	Size (3.3)	Size	3. Size	Size (34)
Brand (28)	4. comfort	Quality (4.5)	Quality	4. Quality	Quality (27)
Quality (31)	5. quality	Brand (4.7)	Brand	5. Type	Brand (26)
Type (35)	6. brand	Type (5.8)	Type	6. Style	Type (19)
Looks (36)	7. looks	Looks (6.0)	Looks	7. Looks	Looks (18)
Style (36)	8. style	Style (6.0)	Style	8. Brand	Style (18)

Explain each group's strategy. What are the relative strengths and weakness of each approach?