INVALIDITY EXAMPLES AND PROBLEMS

"Everything which is known has some justification. Therefore something justifies everything known."

Translation: Jxy : x justifies y; Kx : x is known

$$\frac{(\forall x)(Kx \supset (\exists y)Jyx)}{(\exists y)(\forall x)(Kx \supset Jyx)}$$

First try, one-element domain: D = {a}. The each quantified statement has only one instance, so the premise will be true if and only if Ka ⊃ Jaa. The conclusion will also be true if and only if Ka ⊃ Jaa, so this domain does not allow you to distinguish between the premise and the conclusion. Thus we need to consider a larger domain, {a, b}. In this case, the premise has the instances (Ka ⊃ (∃y)Jya) and (Kb ⊃ (∃y)Jyb). To calculate whether these statement are true (they both must be if the premise is to be true) we need to figure our the right-hand sides by condering their instances, which are Jaa and Jba for the first statement and Jab and Jbb for the second. Since these are instances of an existentially quantified statment, at least one must be true if the existential statement is to be true. Thus we have the following sequence of equivalent conditions:

(∀x)(Kx ⊃ (∃y)Jyx) is true if and only if (Ka ⊃ (∃y)Jya) and (Kb ⊃ (∃y)Jyb are both true,
if and only if (Ka ⊃ (Jaa v Jba)) & (Kb ⊃ (Jab v J bb)) is true.

This last statement express the conditons for truth of the premise (∀x)(Kx ⊃ (∃y)Jyx) in the Domain {a, b}, where each universal quantifier is replaced by an "&" symbol and each existential quantifier is replaced by a "v" symbol.

```
                (∀x)(Kx ⊃ (∃y)Jy
                  .)))))),
(Ka ⊃ (∃y)Jya)  &  (Kb ⊃ (∃y)Jyb)
              +-                 .,
(Ka ⊃ (Jaa v Jba)) & (Kb ⊃ (Jab v J bb))
```

If we perform similar operations on the conclusion of the argument above, we get the following:

```
                (∃y)(∀x)(Kx ⊃ Jyx
                  .)))))),
(∀x)(Kx  ⊃ Jax)  v  (∀x)(Kx  ⊃ Jbx
  *                          .)))))))),
(Ka ⊃ Jaa) & (Kb ⊃ Jab)) v ((Ka ⊃ Jba) & (Kb ⊃  Jbb))
```

Thus for the two element domain {a, b}, we can replace the original argument by:

$$\frac{(Ka \supset (Jaa \vee Jba)) \,\&\, (Kb \supset (Jab \vee J\,bb))}{(Ka \supset Jaa) \,\&\, (Kb \supset Jab)) \vee ((Ka \supset Jba) \,\&\, (Kb \supset Jbb))}$$

This argument is in fact invalid, which you can show by treating each atomic statment Ka, Kb, Jaa, Jab, etc., as if it were a single letter A, B, C,... of propositional logic. If you assign the following truth values to these atomic statement, you will find the required counterexample.

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Ka   Kb   Jaa  Jab  Jba  Jbb
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T T T F F T

This information can be expressed in Teller's format for interpretations as Ka & Kb & Jaa & ~Jab & ~Jba & Jbb, or in standard interpretation format as Interp(K) = {a,b} and Interp(J) = {<a,a>,<b,b>}. Intuitively, this interpretation corresponds to the situation where there are two beliefs a and b, both are known, and both justify themselves. The conclusion is false because no one thing justifies both a and b. If you can find this interpretation or some other counterexample by some other method than that outlined above, more power to you. But the quasi-mechanical method illustrated here is available if you need it.

Practice Problems to Show Invalid:

$(\forall x)(Bx \supset Lxa)$

$(\exists x)(Bx \ \& \ Lxx)$

$(\forall x)Gx \supset (\exists y)Lyy$

$(\forall x)(Gx \supset (\exists y)Lyy)$

Sample Counterexamples:

Domain = {a,b}
~Ba & Bb & ~Laa & Lab & Lba & ~Lbb
(Teller Style)

Domain = {a, b}
Interp(G) = {a}, Interp(L) = {}
(Standard Style)

Practice Problems to Show Invalid:

$(\forall x)(\exists y)Fxy$

$(\exists x)Fxx$

$(\forall x)(\forall y)(Lxy \supset Lyx)$
$(\exists x)(\exists y)Lxy$

$(\exists x)Lxx$

Sample Counterexamples:

Domain = {a,b}
Fab & Fba & ~Faa & ~Fbb

Domain = {a,b}
Interp(L) = {<a,b>,<b,a>}