

**Computer Problem**

1. a. Consider a one-dimensional rod that is insulated along its edges. Assume that it has a length of 10 cm. The rod is initially placed so that one end is  $0^\circ\text{C}$  and the other end is  $100^\circ\text{C}$ . It is allowed to come to a steady-state temperature distribution. Find this temperature distribution,  $u_e(x)$ .

b. At time  $t = 0$ , the one-dimensional rod from Part a is insulated on both ends. This implies that the rod satisfies the PDE:

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} &= \frac{\partial^2 u(x,t)}{\partial x^2}, & t > 0, \quad 0 < x < 10, \\ \text{Boundary Conditions :} & \quad \frac{\partial u(0,t)}{\partial x} = 0, \quad \frac{\partial u(L,t)}{\partial x} = 0, & t > 0, \\ \text{Initial Conditions :} & \quad u(x,0) = u_e(x), & 0 < x < L, \end{aligned}$$

where  $u_e(x)$  is the steady state temperature distribution from Part a. Find the solution to this problem, including the Fourier coefficients. Create a graphic simulation showing the 3D plot of temperature as a function of  $t$  and  $x$ , using 5 and 50 terms (Fourier coefficients) to approximate the solution with  $t \in [0, 20]$ .