

Bessel's Differential Equation - Solutions (exact and series)

```
> de := t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-m^2)*y(t) = 0;
```

$$de := t^2 \left(\frac{d^2}{dt^2} y(t) \right) + t \left(\frac{d}{dt} y(t) \right) + (t^2 - m^2) y(t) = 0 \quad (1)$$

```
> dsolve(de,y(t));
```

$$y(t) = _C1 \text{ BesselJ}(m, t) + _C2 \text{ BesselY}(m, t) \quad (2)$$

```
> Order :=10;
```

$$\text{Order} := 10 \quad (3)$$

```
> dsolve(de,y(t),type=series);
```

$$y(t) = _C1 t^m \left(1 + \frac{1}{-4m-4} t^2 + \frac{1}{(-8m-16)(-4m-4)} t^4 \right. \\ \left. + \frac{1}{(-12m-36)(-8m-16)(-4m-4)} t^6 \right. \\ \left. + \frac{1}{(-16m-64)(-12m-36)(-8m-16)(-4m-4)} t^8 + O(t^{10}) \right) + _C2 t^{-m} \left(1 \right. \\ \left. + \frac{1}{4m-4} t^2 + \frac{1}{(8m-16)(4m-4)} t^4 + \frac{1}{(12m-36)(8m-16)(4m-4)} t^6 \right. \\ \left. + \frac{1}{(16m-64)(12m-36)(8m-16)(4m-4)} t^8 + O(t^{10}) \right) \quad (4)$$

Case when m = 0.

```
> de := t^2*diff(y(t),t$2)+t*diff(y(t),t)+t^2*y(t) = 0;
```

$$de := t^2 \left(\frac{d^2}{dt^2} y(t) \right) + t \left(\frac{d}{dt} y(t) \right) + t^2 y(t) = 0 \quad (5)$$

```
> dsolve(de,y(t));
```

$$y(t) = _C1 \text{ BesselJ}(0, t) + _C2 \text{ BesselY}(0, t) \quad (6)$$

```
> Y := unapply(rhs(%),t);
```

$$Y := t \rightarrow _C1 \text{ BesselJ}(0, t) + _C2 \text{ BesselY}(0, t) \quad (7)$$

```
> dsolve(de,y(t),type=series);
```

$$y(t) = _C1 \left(1 - \frac{1}{4} t^2 + \frac{1}{64} t^4 + O(t^6) \right) + _C2 \left(\ln(t) \left(1 - \frac{1}{4} t^2 + \frac{1}{64} t^4 + O(t^6) \right) \right. \\ \left. + \left(\frac{1}{4} t^2 - \frac{3}{128} t^4 + O(t^6) \right) \right) \quad (8)$$

```
> _C1:=1;_C2:=0;
```

$$_C1 := 1 \quad (9)$$

$$_C2 := 0$$

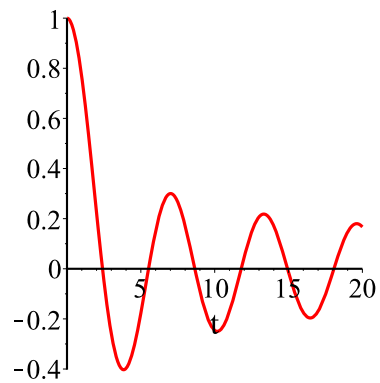
```
> Y(t);
```

$$(10)$$

BesselJ(0, t)

(10)

```
> plot(Y(t), t=0..20);
```



```
> z01:=fsolve(Y(t)=0, t=0..4);
```

$z01 := 2.404825558$

(11)

```
> z02:=fsolve(Y(t)=0, t=4..8);
```

$z02 := 5.520078110$

(12)

```
> _C1:=0;_C2:=1;
```

$_C1 := 0$

(13)

$_C2 := 1$

```
> Y(t);
```

BesselY(0, t)

(14)

```
> plot(Y(t), t=0..20);
```

