Math 531 - Partial Differential Equations

Introduction to Partial Differential Equations

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Outline

Contact Information

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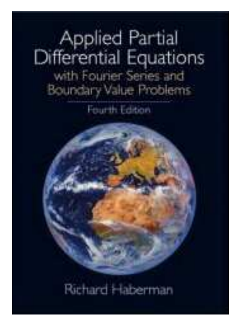
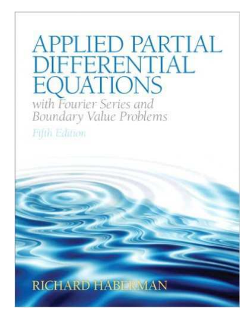
Basic Information: Text

Text: Richard Haberman:

Applied Partial Differential Equations
with Fourier Series and Boundary Value Problems



4^{th} Edition



5^{th} Edition

Basic Information: Topics

- Review Ordinary Differential Equations
- Applications
  - Heat, Laplace's, and Wave Equations
- Primary techniques
  - Separation of Variables/Fourier Series
  - Sturm-Liouville Problems
- Other Problems/techniques
  - Higher Dimensional PDEs
  - Nonhomogeneous Problems
  - Green's Functions
  - Fourier Transforms
  - Method of Characteristics

Prerequisite Courses

- **Math 252: *Calculus III***
  - Series and Integration of Trigonometric Functions
  - Vectors, Partial derivatives, and Gradients
  - Divergence Theorem or Gauss's Theorem
  - Multivariable Integration
- **Math 254: *Linear Algebra***
  - Linear Independence
  - Orthogonality
  - Eigenvalues
- **Math 337: *Ordinary Differential Equations***
  - Existence and Uniqueness of Solutions of ODEs
  - Solutions of Second Order Linear Differential Equations
  - Solving Non-homogeneous ODEs
  - Series Solutions of ODEs
  - Laplace Transforms for Solving ODEs

Basic Information: Grading

Approximate Grading

Homework*	40%
Exams and Final×	60%

* Written HW, which includes problems from WeBWorK. Lecture worksheets, showing lecture understanding. Some exercises will include **MatLab** and/or **Maple** programs.

× Likely to be 2 Midterms and Final with half being Take-home. Final: Monday, May 8, 13:00 – 15:00, possibly take-home.

Expectations and Procedures, I

- Most class attendance is OPTIONAL — Homework and announcements will be posted on the class web page.
  If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn off cell phones.
  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.

Expectations and Procedures, II

- Please, turn in assignments on time. (The instructor reserves the right not to accept late assignments.)
- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. ***Please contact the instructor EARLY regarding special circumstances.***
- Students are expected ***and encouraged*** to ask questions in class!
- Students are expected ***and encouraged*** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

Expectations and Procedures, III

- Missed midterm exams: Don't miss exams! The instructor reserves the right to schedule make-up exams and/or base the grade solely on other work (including the final exam).
- Missed final exam: Don't miss the final! Contact the instructor ASAP or a grade of incomplete or F will be assigned.
- ***Academic honesty***: Submit your own work. Any cheating will be reported to University authorities and a **ZERO** will be given for that HW assignment or Exam.

MatLab/Maple Programs

Some Programming in **MatLab** and/or **Maple**

- Students can obtain **MatLab** from EDORAS Academic Computing – Google **SDSU MatLab** or access **http://edoras.sdsu.edu/∼download/matlab.html**
- **MatLab** and **Maple** can also be accessed in the **Computer Labs GMCS 421**, **422**, and **425?**.
- I may inquire about special deals through the **Maple adoption** program if there is sufficient interest in class.

What is a Partial Differential Equation (PDE)?

Ordinary Differential Equation (ODE) – Studied in Math 337 (or equivalent Math 342A or AE 280)
Typically, an ODE can be written

$$\frac{dy}{dt} = f(t,y),$$

where $y(t)$ is an unknown function and may be a vector in $\mathbb{R}^n$

Partial Differential Equation (PDE) is an equation of an unknown function $u(t,\tilde{\mathbf{x}})$ that includes partial derivatives of this unknown function.
Often, u is a scalar quantity, *e.g.*, temperature, t is time, and $\tilde{\mathbf{x}} \in \mathbb{R}^n$

Heat Equation: Let $u(t,x)$ be temperature in a rod:

$$\frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2}, \qquad t > 0, \quad 0 < x < L.$$

Math 531: Learning Objectives for PDEs

Learning Objectives for Partial Differential Equations (PDEs)

1. Connect significant physical problems with PDEs
2. Learn tools for solving PDEs, including visualization through programming
3. Manage the methods and details for large multi-step problems
4. Explore decomposition of continuous functions with Fourier series
5. Develop intuition for extending finite dimensional vector spaces (254/524) to infinite dimensions
6. Appreciate the complexities and varied techniques for PDEs

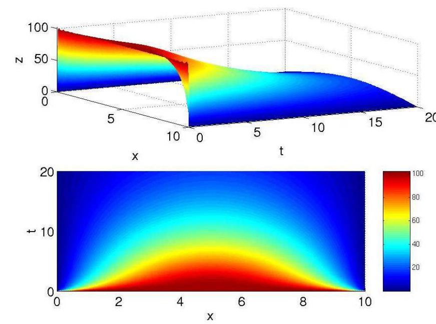
Heat Equation in a Rod

Heat Equation in a Rod: Let $z(t,x)$ be temperature in a rod:

$$\frac{\partial z(t,x)}{\partial t} = \frac{\partial^2 z(t,x)}{\partial x^2}, \qquad t > 0, \quad 0 < x < 10.$$

Initial and boundary conditions:

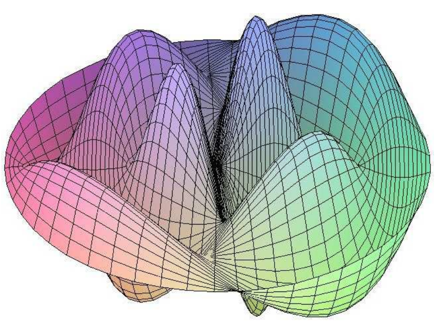
$$z(0,x) = 100, \qquad z(t,0) = 0 = z(t,10).$$



Vibrations on a Circular Membrane

Vibrations on a Circular Membrane: Let $u(t,r,\theta)$ be displacement of a circular membrane:

$$\frac{\partial^2 u}{\partial t^2} = c^2\nabla^2 u, \qquad t > 0, \quad 0 < r < 1, \quad -\pi < \theta \le \pi.$$



Maple Worksheet – Vibration

More Partial Differential Equations

Laplace's Equation or Steady-State: Let $u(x,y,z)$ be temperature in a rectangular box in $\mathbb{R}^3$:

$$\nabla^2 u = 0, \qquad 0 < x < a, \quad 0 < y < b, \quad 0 < z < c.$$

Reaction-Diffusion Equation: Let $c(t,x,y,z)$ be the concentration in a region $R \in \mathbb{R}^3$, D be diffusivity, and $f(c)$ represent a chemical reaction:

$$\frac{\partial c}{\partial t} = f(c) + \nabla\cdot(D\nabla c), \qquad t > 0, \qquad (x,y,z) \in R.$$

More Partial Differential Equations

Age-structured model or McKendrick/von Foerster equation:
Let $p(t,a)$ be the population in time t with individual ages a:

$$\frac{\partial p}{\partial t} + V(p)\frac{\partial p}{\partial a} = r(t,p), \qquad t > 0, \quad a > 0.$$

Nonlinear waves - Korteweg-deVries: Let $u(t,x)$ be the wave height in shallow water:

$$\frac{\partial u}{\partial t} + (w'(0) + \beta u)\frac{\partial u}{\partial x} = \frac{w'''(0)}{3!}\frac{\partial^3 u}{\partial x^3}, \qquad t > 0.$$

Schrödinger Equation: Let $A(t,x)$ be the amplitude of the wave height for monochromatic light:

$$\frac{\partial A}{\partial t} + w'(k_0)\frac{\partial A}{\partial x} = i\frac{w''(k_0)}{2!}\frac{\partial^2 A}{\partial x^2}, \qquad t > 0.$$