February 17, 2017 Math 124a Name Key

1. (10 pts) During the human female menstrual cycle, the gonadotropin, FSH or follicle stimulating hormone, is released from the pituitary in a sinusoidal manner with a period of approximately 28 days. Guyton's text on *Medical Physiology* shows that if we define day 0 ($t = 0$) as the beginning of menstruation, then FSH, $F(t)$, cycles with a high concentration of about 4 ("relative units") around day 9 and a low concentration of about 1.5 around day 23.

a. Consider a model of the concentration FSH (in "relative units") given by

$A = \frac{1.5+4}{2} = 2.75$

$B = 1.25$

$$F(t) = A + B\cos(\omega(t - \phi)),$$

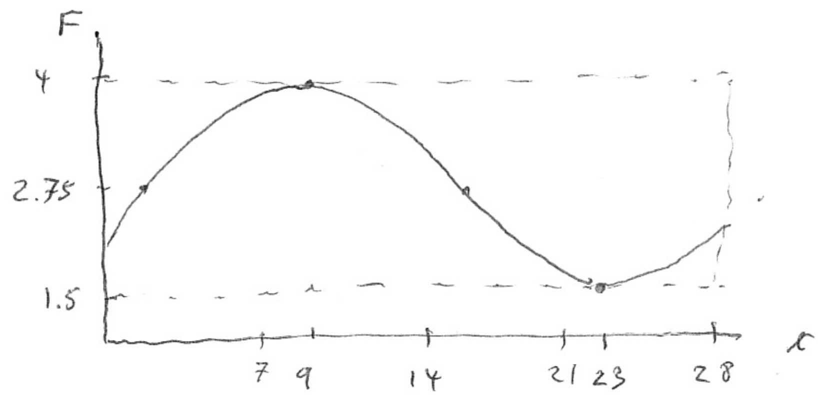
where A, B, ω, and ϕ are constants and t is in days. Use the data above to find the four parameters, then sketch a graph for the concentration of FSH over one period. If ovulation occurs around day 14, then what is the approximate concentration of FSH at that time?

$\omega = \frac{2\pi}{28}$

$A =$ 2.75 $B =$ 1.25 $\omega = \frac{\pi}{14} = 0.2244$

$\phi =$ 9 $F(14) =$ 3.2924

GRAPH:



b. Create an equivalent model in the form:

$$G(t) = C + D\sin(\nu(t - \psi)),$$

with $\psi \in [0, T)$, where T is the period of the function.

$C =$ 2.75 $D =$ 1.25 $\nu =$ 0.2244

$\psi =$ 2

$\frac{\pi}{14}(9 - \psi) = \frac{\pi}{2} \Rightarrow 9 - \psi = 7$

$\psi = 2$

2. (10 pts) a. *Staphylococcus aureus* is a common cause of skin infections and can lead to serious complications in hospitals, including death (MRSA). A common means of measuring populations of bacteria is through optical density (OD_{650}). Suppose a culture satisfies the Malthusian growth:

$$P_{n+1} = (1+r)P_n,$$

where n is in minutes. If the initial OD_{650} is 0.043, *i.e.*, $P_0 = 0.043$, and after 25 min, $P_{25} = 0.071$, then find the value of r. Determine the doubling time for this culture and estimate the OD_{650} reading at 60 min (P_{60}), assuming continued Malthusian growth. (Give all numbers to at least **4 significant figures**.)

$P_n = 0.043(1+r)^n$ $\qquad 0.071 = 0.043(1+r)^{25}$ $\qquad 1+r = \left(\frac{71}{43}\right)^{1/25}$

$t_d = \dfrac{\ln(2)}{\ln(1.020263)}$

$r =$ 0.020262 $\qquad P_{60} =$ 0.14327 OD_{650}

Doubling time = 34.555 min

b. A mutant strain also grows according to a Malthusian growth law:

$$M_{n+1} = (1+s)M_n.$$

Assume this culture has a doubling time of 31 min and begins with less than 10% of the population, OD_{650} is 0.004, or $M_0 = 0.004$. Determine the value of s and find a general expression for M_n.

$M_n = 0.004(1+s)^n$ $\qquad 2 = (1+s)^{31}$ $\qquad 1+s = 2^{1/31}$

$s =$ 0.022611 $\qquad M_n =$ $0.004(1.022611)^n$ OD_{650}

c. Assuming these cultures start at the same time, find how long it takes for them to have the same OD_{650} reading.

$P_m = M_m \Rightarrow 0.043(1.020262)^m = 0.004(1.022611)^m$

$\frac{43}{4} = \left(\frac{1.022611}{1.020262}\right)^m$

$P_m = M_m$, when $m =$ 1032.39 min $\qquad m = \dfrac{\ln(43/4)}{\ln\left(\frac{1.022611}{1.020262}\right)}$