January 27, 2017 Math 124a Name Key

1. (5pts) a. The rate at which a cricket chirps has been shown to vary almost linearly with the temperature. Thus, an appropriate model has the form

$$T(N) = mN + b,$$

where N is the number of chirps/min of a cricket and T is the temperature in degrees Fahrenheit (°F). Suppose that a cricket is found be chirping at a rate of 63 chirps/min when the temperature is 54°F, while it chirps at a rate of 177 chirps/min when the temperature is 86°F. Use the data above to find the values of m and b.

$$m = \frac{86-54}{177-63} = \frac{16}{57}$$

$m =$ 0.2807 $b =$ 36.32

$$b = 54 - m \cdot 63$$

b. Use this model to estimate the temperature when a cricket is chirping at a rate of 96 chirps/min. Also, approximate the chirp rate when the environmental temperature is 72°F.

Temperature for 96 chirps/min 63.26°F Chirp rate when 72°F 127.1 chirp/min

2. (5 pts) a. Populations of animals that reproduce seasonally often satisfy a discrete population model. One of the most common models of this type used in biology is the logistic growth model. Suppose a population satisfies the model given by:

$$P_{n+1} = P_n + g(P_n) = P_n + 0.04\, P_n \left(1 - \frac{7P_n}{237}\right),$$

where P_n is the population (in thousands) in the n^{th} season.

a. This model has a quadratic growth rate:

$$g(P) = 0.04\, P \left(1 - \frac{7P}{237}\right).$$

The population is at equilibrium when the growth rate is zero, so determine the equilibrium populations by solving $g(P) = 0$.

$$\Rightarrow P = 0, \quad 1 - \frac{7P}{237} = 0 \text{ or } P_e = \frac{237}{7}$$

Equilibrium populations = 0, 33.857

b. The growth rate is at a maximum at the vertex of parabola. Find the population that produces this maximum growth rate, P_{max}, and what that growth rate is, $g(P_{max})$.

$$P_{max} = P_v = \frac{33.857}{2}$$

$P_{max} =$ 16.929 $g(P_{max}) =$ 0.3386

3. (10pts) Consider the curves

$$\frac{-1 \pm \sqrt{1+12}}{2}$$

$$f(x) = -2x - 3 \qquad \text{and} \qquad g(x) = x^2 + x - 3.$$

Find all the x and y-intercepts for both curves. Determine the slope of the line and the vertex of the parabola. Find the points of intersection, then sketch the graph of these curves. (Label the points clearly.)

Line: x-intercept $-3/2$ y-intercept -3 Slope $m =$ -2

Parabola: x-intercepts $-\frac{1}{2} \pm \frac{\sqrt{13}}{2}$ y-intercept -3 Vertex $(-\frac{1}{2}, -3\frac{1}{4})$

-2.303 or 1.303

Points of Intersection: $(-3, 3)$ and $(0, -3)$

$x^2 + 3x = 0$

$x = 0, -3$

GRAPH: