February 22, 2017 Math 124a Name Key

For **FULL CREDIT** you need to show how you obtained your answer. You can staple **approved** scratch paper if necessary.

1. (20pts) For the following functions, find the domain. Find all x and y-intercepts. Determine any horizontal or vertical asymptotes. (**If any intercept or asymptote fails to exist, then write "NONE."**) Sketch the graphs of the functions.

a. $y = 8 - 3\,e^{-x/3}$. $8 = 3e^{-x/3}$ $e^{x/3} = 3/8$ $x = 3\ln(3/8)$

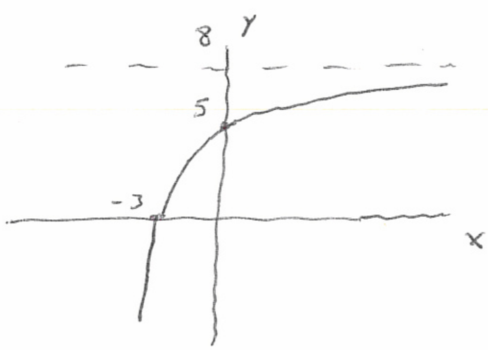
1 Domain: All x

3, 1 x-intercept -2.9425 and y-intercept 5

1 Vertical Asymptotes: NONE

2 Horizontal Asymptotes: $y = 8$ as $x \to +\infty$

GRAPH:



2

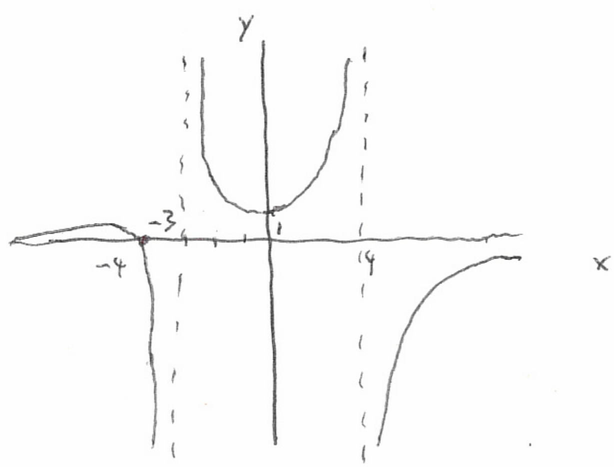
b. $y = \dfrac{3x+12}{12+x-x^2}$. $x^2 - x - 12 = (x-4)(x+3) = 0$

1, 1, 1 Domain: $x \neq -3, 4$ x-intercept(s) -4 y-intercept 1

2, 2 Vertical Asymptotes: $x = -3, 4$ Horizontal Asymptotes: $y = 0$

GRAPH:

3



20

2. (14pts) a. Animals shake their coats to remove water (and prevent getting too cold)[1]. A 0.31 kg rat (*Rattus norvegicus*) shakes with a characteristic frequency of 17.9 Hz, while a 90.0 kg Black bear (*Ursus americanus*) shakes with a characteristic frequency of 4.1 Hz. An allometric model for the frequency (F) in Hz as a function of weight (W) in kg satisfies the relationship given by

$$F = aW^k, \quad \ln(F) = \ln(a) + k\ln(W)$$

for some constants a and k. Find the constants a and k.

W	$\ln(W)$	F	$\ln(F)$
0.31	−1.17118	17.9	2.88480
90	4.49981	4.1	1.41099

$$k = \frac{\ln(F_2) - \ln(F_1)}{\ln(W_2) - \ln(W_1)} = \frac{1.41099 - 2.88480}{4.49981 + 1.17118}$$

$$\ln(a) = \ln(F_1) - k\ln(W_1)$$
$$= 2.88480 + 0.25989(-1.17118) = 2.58043$$

4, 4 $a =$ 13.2028 $k =$ −0.25989

b. A river otter (*Amblonyx cinereus*) weighs 3.5 kg. Use the above model to predict the frequency that this otter will shake. If it actually shakes with a frequency of 10.2 Hz, then determine the percent error from the model (assuming that the actual frequency is the best).

$$F = 13.2028\,(3.5)^{-0.25989} = 9.5339$$

$$Err = \frac{(9.5339 - 10.2)}{10.2} \cdot 100$$

2.5 Model $F =$ 9.5339 Hz Percent Error = −6.53%

c. If the frequency of the shake of a guinea pig (*Cavia porcellus*) is measured to be 14.1 Hz, then use the model to predict the weight, W, of the grey squirrel. If it actually has a weight of 0.61 kg, then determine the percent error from the model (assuming that the actual weight is the best).

$$14.1 = 13.2028\,W^{-0.25989}$$

$$W = \left(\frac{14.1}{13.2028}\right)^{\frac{1}{-0.25989}}$$

$$Err = \frac{(0.77648 - 0.61)}{0.61} \cdot 100$$

3.5 Model $W =$ 0.77648 kg Percent Error = 27.29%

[1] A. K. Dickerson, Z. G. Mills and D. L. Hu, (2012) Wet mammals shake at tuned frequencies to dry, *J. R. Soc. Interface*, doi: 10.1098/rsif.2012.0429

14

3. (18pts) a. The body temperature of a woman varies over her period in a sinusoidal manner. A reasonable model for one woman's body temperature satisfies the following:

$$T(t) = 36.5 - 1.5\cos\left(\frac{\pi}{14}(t-8)\right),$$

$P = \frac{2\pi}{\pi/14} = 28$

where t is in days and T is the body temperature in C°. Find the period, amplitude, phase shift, and vertical shift for this model. Give the t and T values for the maximum body temperature $(t_{max}, T(t_{max}))$ and minimum body temperature $(t_{min}, T(t_{min}))$ for $t \in [0, P]$, where P is the length of the period. Sketch the graph of this model for $t \in [0, P]$.

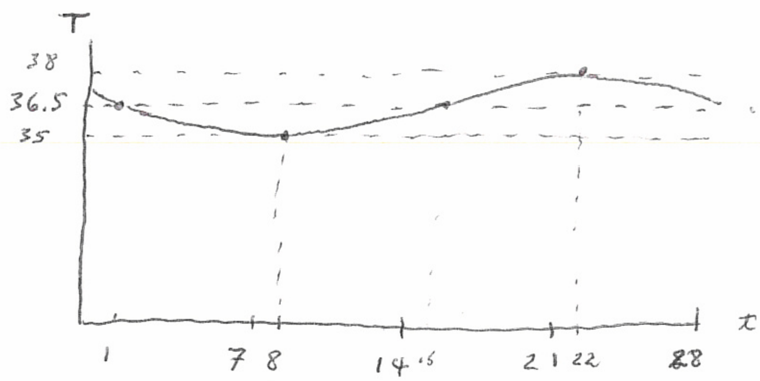
2, 1 Period $(P) =$ 28 Amplitude $=$ 1.5

1, 1 Phase Shift $=$ 8 Vertical Shift $=$ 36.5

2, 2 $(t_{max}, P(t_{max})) =$ (22, 38) $(t_{min}, P(t_{min})) =$ (8, 35)

GRAPH:

3



b. Create an equivalent model in the form:

$$T(t) = 36.5 + 1.5\cos\left(\frac{\pi}{14}(t-\phi)\right),$$

with $\phi \in [0, P)$, where P is the period of the model.

Max at $t = 22 \Rightarrow \phi = 22$

3 $\phi =$ 22

c. Next create an equivalent model in the form:

$$T(t) = 36.5 + 1.5\sin\left(\frac{\pi}{14}(t-\psi)\right),$$

with $\psi \in [0, P)$, where P is the period of the model.

Max at 22

3 $\psi =$ 15

$\frac{\pi}{14}(22-\psi) = \frac{\pi}{2}$

$22 - \psi = 7$

$\psi = 15$

4. (17pts) a. If we let C_n be the concentration of a drug (in ng/ml of blood) after n hr and α be the rate of metabolism and excretion of the drug, then an appropriate discrete model satisfies the equation:

$$C_{n+1} = (1-\alpha)C_n.$$

Suppose the drug is injected into a test subject, and soon afterwards the blood concentration is measured, giving $C_0 = 23.0$ ng/ml of blood. Another blood sample is drawn 20 hr later shows the concentration has dropped to 12.5 ng/ml of blood. Find the kinetic rate constant α and determine the half-life of this drug in this subject.

$C_n = C_0(1-\alpha)^n$ $\qquad$ $C_{20} = 12.5 = 23(1-\alpha)^{20}$

$1-\alpha = \left(\frac{12.5}{23}\right)^{1/20}$

$n_h = \frac{\ln(1/2)}{\ln(1-\alpha)}$

4, 2 $\quad \alpha =$ 0.030028 $\quad$ Half-life = 22.735 hr

b. To be beneficial the drug needs to be administered regularly. In addition, this allows lower doses to decrease toxicity of the drug. A new model, which includes both the daily loss of the drug from metabolism and excretion and the regular administration of the drug, μ, satisfies the equation

$$D_{m+1} = (1-\beta)D_m + \mu,$$

where D_m is the concentration of the drug in ng/ml of blood, β is the decay rate, and m is in units of days. Below is a table showing the drug concentration for three successive days.

Day	0	1	2
D_m ng/ml	10.1	15.8	20.1

Use the data above to find the constants, β and μ. Determine the concentration of the drug for the next two days, D_3 and D_4.

$20.1 = (1-\beta)15.8 + \mu$
$15.8 = (1-\beta)10.1 + \mu$
$4.3 = (1-\beta)5.7$

$1-\beta = \frac{4.3}{5.7} \Rightarrow \beta = 1 - \frac{4.3}{5.7}$

$\mu = 15.8 - 0.75439(10.1)$

3, 2 $\quad \beta =$ 0.24561 $\quad \mu =$ 8.18070

1, 1 $\quad D_3 =$ 23.3439 ng/ml $\quad$ and $\quad D_4 =$ 25.7910 ng/ml

c. Find the equilibrium concentration of the drug in this subject's blood based on the model in Part b. What is the stability of this equilibrium concentration?

$D_e = (1-\beta)D_e + \mu \Rightarrow \beta D_e = \mu$ or $D_e = \mu/\beta$

3, 1 $\quad$ Equilibrium $D_e =$ 33.3071 $\quad$ STABLE or UNSTABLE (Circle one)

17

5. (16pts) As girls age their height and weight grow at differing rates. Below we present a table of the heights and weights of girls ages 6, 9, and 11.

Weight, w (kg)	20.2	29.3	37.6
Height, h (cm)	114	132	144

a. A best fitting linear model for the height as a function of weight is:

$$h(w) = 1.73\,w + 79.8.$$

When using this function, compute the model, h, values to **5 significant figures.** Use the data from the table above to write all the square errors, then compute the sum of square errors.

$h(20.2) = 114.746$ $\quad$ $h(29.3) = 130.489$ $\quad$ $h(37.6) = 144.848$

5 $\quad$ $e_1^2 =$ 0.5565 $\quad$ $e_2^2 =$ 2.2831 $\quad$ $e_3^2 =$ 0.7191

Sum of Square Errors = 3.5587

b. Ehrenberg found that the model

$$h(w) = 48.3\,\ln(w) - 31.1,$$

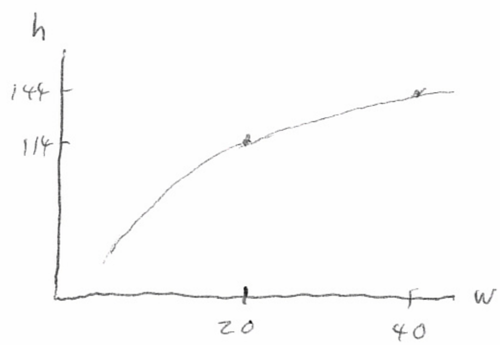
fit the data over a range of ages very well. Find the height at $w = 29.3$ using both models and determine the percent error for each model (assuming the data in the Table is the best). Sketch a graph of this model.

1, 1 $\quad$ Linear Model: $h =$ 130.489 cm $\quad$ Percent error = −1.145 %

1, 1 $\quad$ Ehrenberg Model: $h =$ 132.0375 cm $\quad$ Percent error = 0.0284 %

GRAPH

2



c. Use both models to predict the weight of a girl with a height of 150 cm.

$150 = 1.73 \cdot w + 79.8$

2 $\quad$ Linear Model: $w =$ 40.578 kg $\quad$ $w = \dfrac{70.2}{1.73}$

3 $\quad$ Ehrenberg Model: $w =$ 42.499 kg

$150 = 48.3 \ln(w) - 31.1$

$\ln(w) = \dfrac{181.1}{48.3}$

16