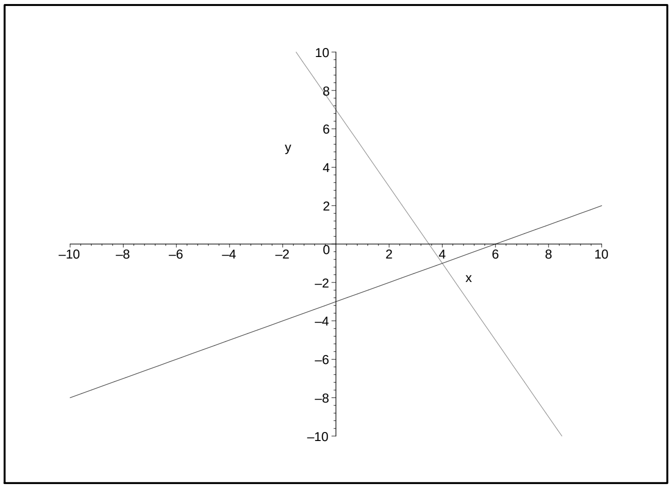
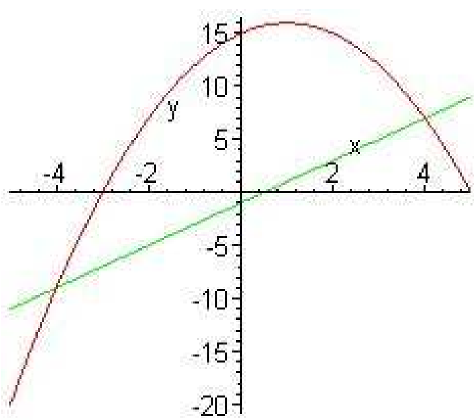
1. The first line is $y = \frac{x}{2} - 3$. The perpendicular line $y = -2x + 7$. The graph of these two lines is below.

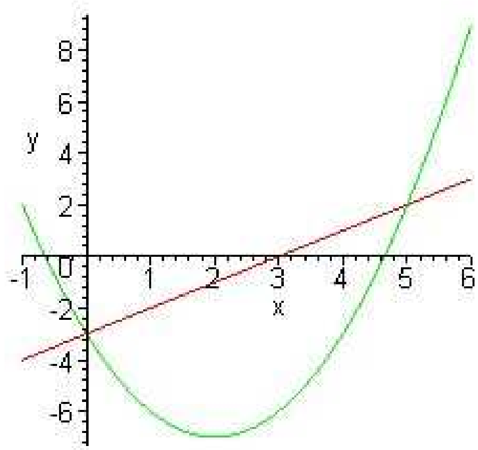


2. The weight of the dog is 19.5 kg, and its temperature 38.9°C.

3. For $f(x) = 2x - 1$, the x and y-intercepts are $(\frac{1}{2}, 0)$ and $(0, -1)$, respectively. The slope is $m = 2$. For $g(x) = 15 + 2x - x^2$, the x-intercepts are $x = 5, -3$, and the y-intercept is $(0, 15)$. The vertex is $(1, 16)$. The curves intersect at $(-4, -9)$ and $(4, 7)$. The graph of the line and the parabola are below to the left.



Problem 3

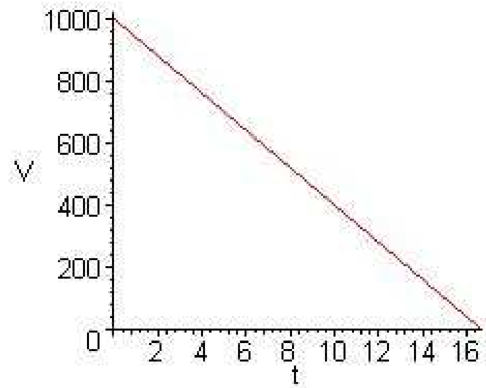


Problem 4

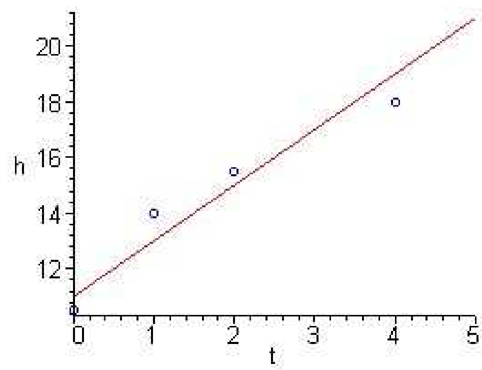
4. For $f(x) = x - 3$, the x and y-intercepts are $(3, 0)$ and $(0, -3)$, respectively. The slope is $m = 1$. For $g(x) = x^2 - 4x - 3$, the x-intercepts are $x = 2 \pm \sqrt{7}$, and the y-intercept is $(0, -3)$. The vertex is $(2, -7)$. The curves intersect at $(0, -3)$ and $(5, 2)$. The graph of the line and the parabola are above to the right.

5. The linear model satisfies $V(t) = 1000 - 60t$ The water is lost at time $t = \frac{1000}{60} \simeq 16.7$ weeks.

Note that this model is only valid for $0 \leq t \leq \frac{100}{6}$. The graph of the water evaporating is shown below to the left.



Problem 5



Problem 6

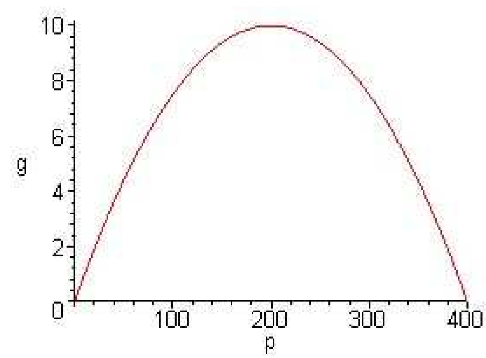
6. a. The sum of the squared error is given by $J = 2.5$. The graph of the model with the data points is above to the right. Over the given domain the model is reasonable. However, this model predicts that the plant will keep growing at a constant rate, which is not commonly observed.

b. The model predicts the height of the plant at 3 and 5 weeks to be $h(3) = 17$ cm and $h(5) = 21$ cm.

7. a. The equilibrium populations are $P_e = 0$ and 400.

b. The growth function $g(p)$ has maximum growth rate at the population $P = 200$ with $g(200) = 10$ individuals per generation.

c. The graph $g(P) = 0.1P(1 - \frac{P}{400})$ is shown below.



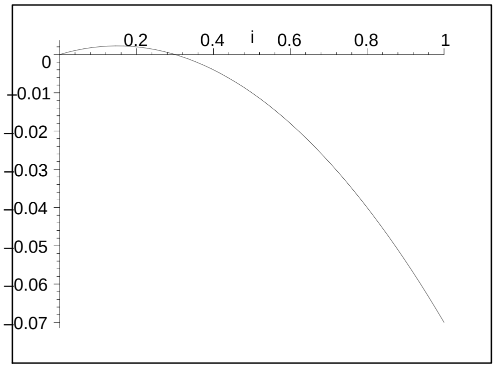
8. a. The equilibrium are found by solving

$$F(i) = 0.1i(1 - i) - 0.07i = 0.03i - 0.1i^2 = 0.1i(0.3 - i) = 0.$$

Thus, equilibria are given by $i_e = 0$ and $i_e = 0.3$

b. The infection function $F(i)$ has a maximum infection rate at $i_v = 0.15$ with $F(0.15) = 0.00225$.

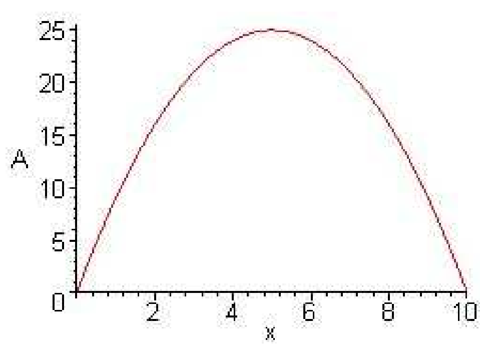
c. We find $F(1) = -0.07$, and the graph of $F(i) = 0.1i(0.3 - i)$ is shown below.



9. a. A rectangle with length x, width y, and a perimeter of 20 cm, has its width satisfy $y = 10 - x$.

b. The area of a rectangle is $A(x) = x(10 - x) = 10x - x^2$. The domain is given by $0 \leq x \leq 10$.

c. $A(x)$ is a parabola, and its graph is shown below. The vertex occurs at $x = 5$ with $A(5) = 25$ cm^2. Thus, the particular rectangle with the largest area is a square that measures 5 cm on a side.



10. a. If $e^a = 2.2$ and $e^b = 0.7$, then

$$\frac{e^{a+b}}{(e^b + e^0)^2} = \frac{154}{289} \simeq 0.5329.$$

10. b. If $e^a = 2.2$ and $e^b = 0.7$, then

$$\frac{(e^a)^2(e^0 - e^b)}{e^{2a+b}} = \frac{3}{7} \simeq 0.429.$$

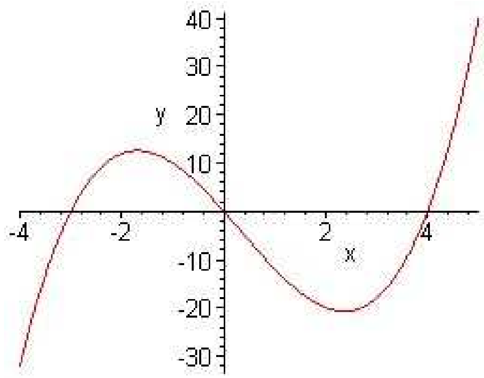
10. c. If $\ln(c) = 1.3$ and $\ln(d) = -0.5$, then

$$\frac{\ln(c^3) - \ln(c) + \ln(1)}{(\ln(c) + \ln(e))} = \frac{26}{23} \simeq 1.13.$$

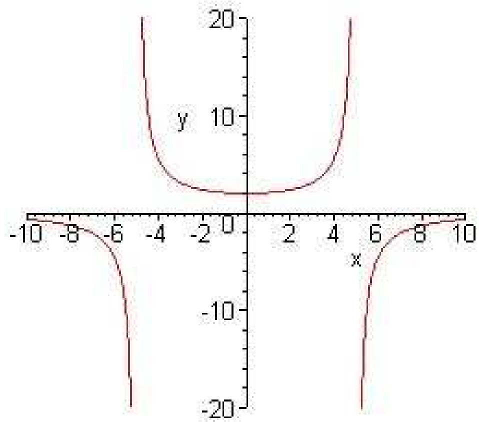
10. d. If $\ln(c) = 1.3$ and $\ln(d) = -0.5$, then

$$\frac{\ln(c^2 d) - \ln(1)}{(\ln(c/d) - \ln(e))} = \frac{21}{8} \simeq 2.625.$$

11. a. For $y = x^3 - x^2 - 12x$, the domain is all x. The x-intercepts are $(-3, 0)$, $(0, 0)$, and $(4, 0)$. The y-intercept is $(0, 0)$. There are no asymptotes. The graph is below to the left.



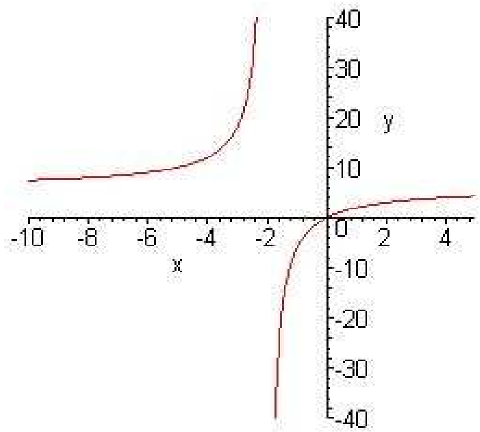
Problem 11.a.



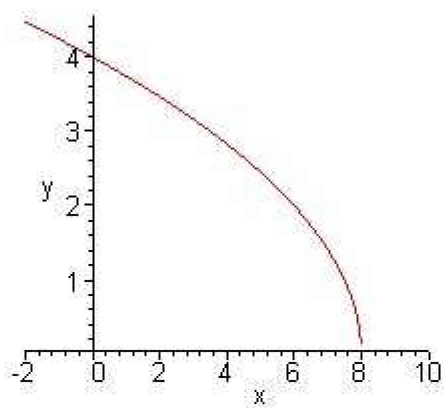
Problem 11.b.

11. b. For $y = \dfrac{50}{25 - x^2}$, the domain is $x \neq \pm 5$. There is no x-intercept. The y-intercept is $(0, 2)$. The vertical asymptotes satisfy $x = \pm 5$, and the horizontal asymptote is $y = 0$. The graph is above to the right.

11. c. For $y = \dfrac{6x}{x + 2}$, the domain is $x \neq -2$. The x and y-intercept is $(0, 0)$. The vertical asymptote is $x = -2$, while the horizontal asymptote is $y = 6$. The graph is below to the left.



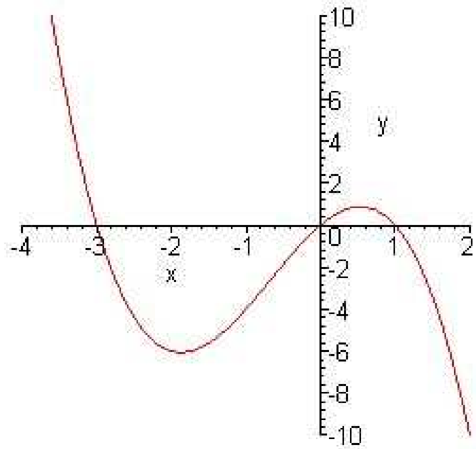
Problem 11.c.



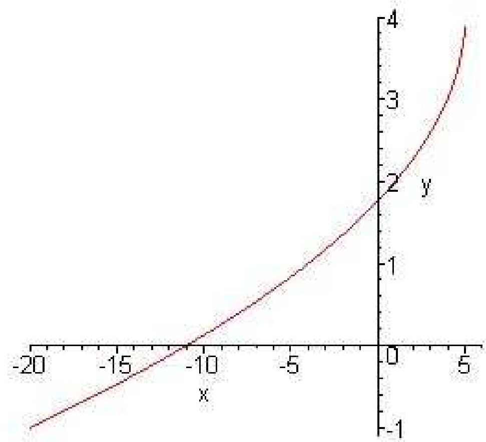
Problem 11.d.

11. d. For $y = \sqrt{16 - 2x}$, the domain is $x \leq 8$. The x-intercept is $(8, 0)$. The y-intercept is $(0, 4)$. There are no asymptotes. The graph is above to the right.

11. e. For $y = 3x - 2x^2 - x^3$, the domain is all x. The x and y-intercepts are $(-3, 0)$, $(0, 0)$ and $(1, 0)$. There are no asymptotes. The graph is below to the left.



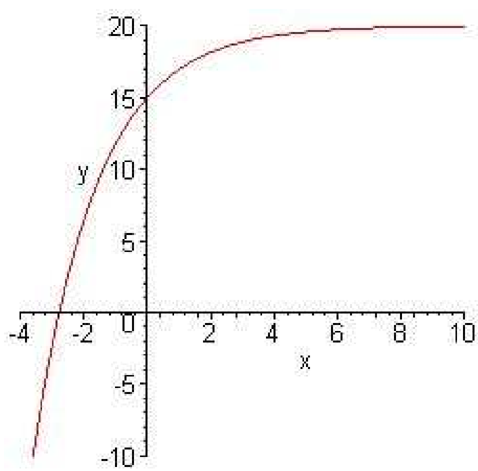
Problem 11.e.



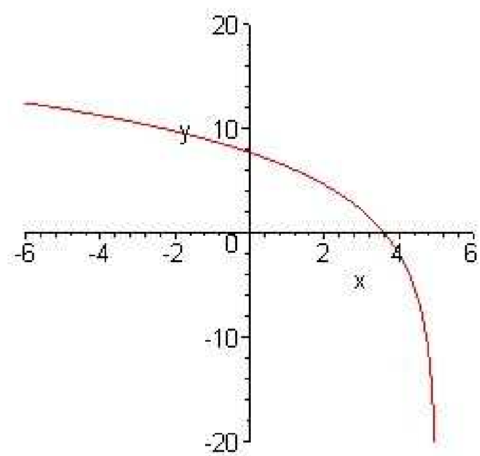
Problem 11.f.

11. f. For $y = 4 - \sqrt{5 - x}$, the domain is $x \leq 5$. The x and y-intercepts are $(-11, 0)$ and $(0, 1.7639)$. There are no asymptotes. The graph is above to the right.

11. g. For $y = 20 - 5e^{-0.5x}$, the domain is all x. The x and y-intercepts are $(-2.773, 0)$ and $(0, 15)$, respectively. The horizontal asymptote as $x \to \infty$ is $y = 20$. The graph is below to the left.



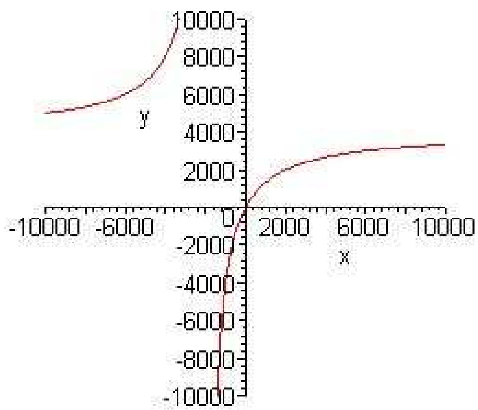
Problem 11.g.



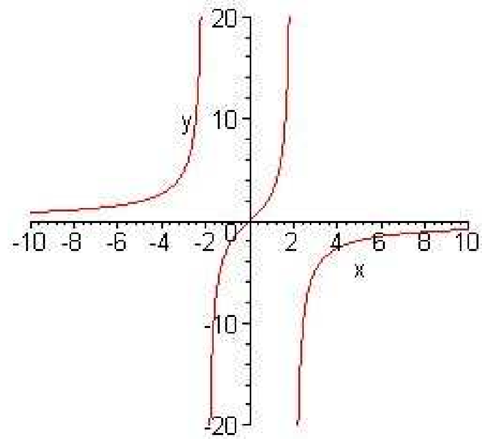
Problem 11.h.

11. h. For $y = 6\ln(5 - x) - 2$, the domain is $x < 5$. The x and y-intercepts are $(3.604, 0)$ and $(0, 7.657)$, respectively. The vertical asymptote is $x = 5$ with $y \to -\infty$. The graph is above to the right.

11. i. For $y = \dfrac{4x}{2 + 0.001x}$, the domain is $x \neq -2000$. The vertical asymptote is $x = -2000$. The x and y-intercept is $(0, 0)$. The horizontal asymptote is $y = 4000$. The graph is below to the left.



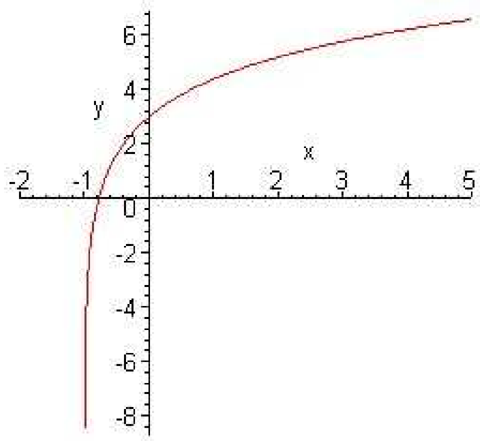
Problem 11.i.



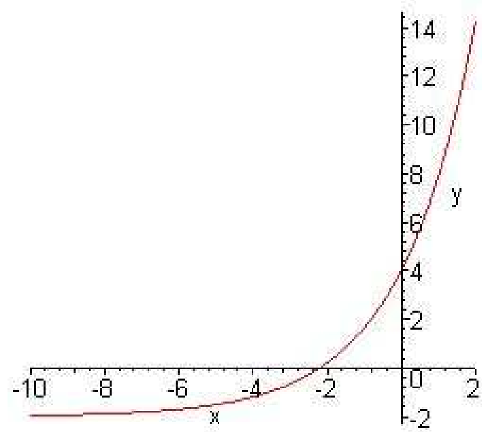
Problem 10.j.

11. j. For $y = \dfrac{8x}{4 - x^2}$, the domain is $x \neq \pm 2$. The vertical asymptotes are $x = \pm 2$. The x and y-intercept is $(0, 0)$. A horizontal asymptote is $y = 0$ as $x \to \infty$. The graph is above to the right.

11. k. For $y = 3 + 2\ln(x + 1)$, the domain is $x > -1$. The x and y-intercepts are $(-0.7769, 0)$ and $(0, 3)$. The vertical asymptote is $x = -1$ with $y \to -\infty$. The graph is below to the left.



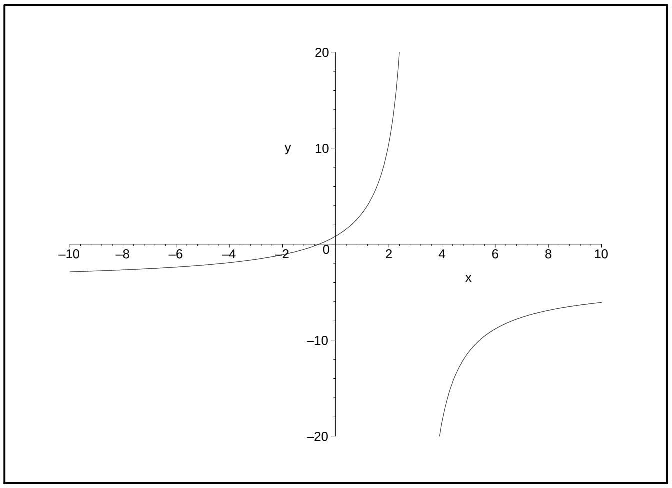
Problem 11.k.



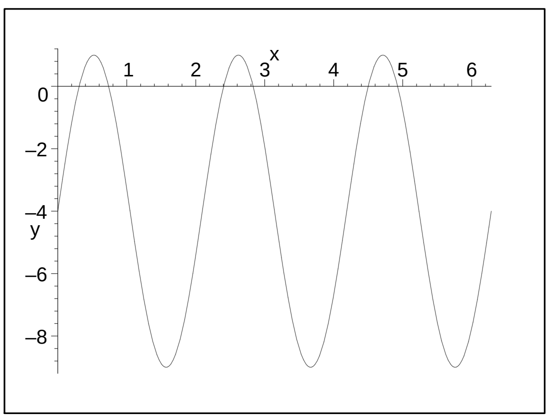
Problem 11.l.

11. l. For $y = 6e^{x/2} - 2$, the domain is all x. The x and y-intercepts are $(-2.197, 0)$ and $(0, 4)$. The horizontal asymptote as $x \to -\infty$ is $y = -2$. The graph is above to the right.

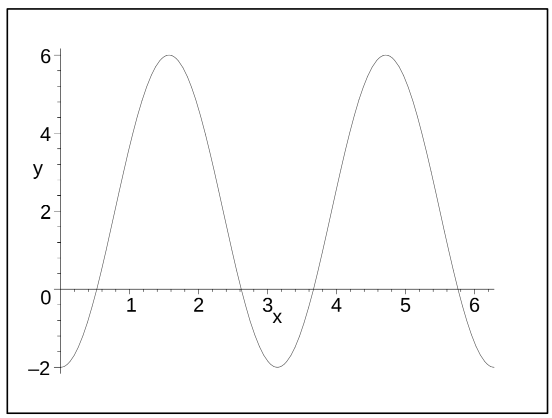
11. m. For $y = \frac{8x+5}{6-2x}$, the domain is $x \neq 3$. The x and y-intercepts are $\left(-\frac{5}{8}, 0\right)$ and $\left(0, \frac{5}{6}\right)$. The vertical asymptote is $x = 3$. The horizontal asymptote is $y = -4$. The graph is below.



12. The function, $y = 5\sin(3x) - 4$, has a period of $x = 2\pi/3$. The function oscillates about $y = -4$, the vertical shift, with an amplitude of 5. It begins at $(0, -4)$, goes to a maximum at $(\pi/6, 1)$, continues through $(\pi/3, -4)$, then reaches a minimum at $(\pi/2, -9)$, and ends its cycle at $(2\pi/3, -4)$. The maxima occur at $x = \pi/6, 5\pi/6, 3\pi/2$. The graph of the function is below.



Problem 12



Problem 13

13. a. The function, $y = 2 - 4\cos(2x)$, has a period of $x = \pi$. The function oscillates about $y = 2$ with an amplitude of 4. It begins at a minimum at $(0, -2)$, goes to a maximum at $(\pi/2, 6)$, then ends its cycle at $(\pi, -2)$. The maxima occur at $x = \pi/2, 3\pi/2$ with y values of 6. The graph of the function is above.

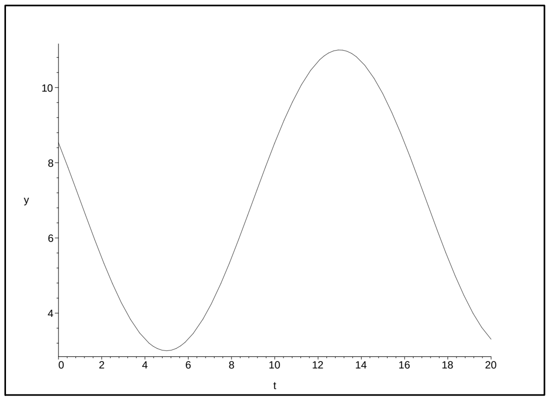
b. The equivalent form has $A = 2$, $B = 4$, and $\omega = 2$. Since the amplitude has a negative sign, we phase shift the function by half a period or $\phi = \frac{\pi}{2}$. Thus,

$$y(x) = 2 + 4\cos\left(2\left(x - \frac{\pi}{2}\right)\right).$$

c. The notes show that the equivalent sine model is shifted a quarter period from the cosine model in Part b. Thus, $C = A$, $D = B$, $\nu = \omega$, and $\psi = \phi - \frac{\pi}{4}$ or

$$y(x) = 2 + 4\sin\left(2\left(x - \tfrac{\pi}{4}\right)\right).$$

14. a. The function, $y(t) = 7 - 4\cos\left(\frac{\pi}{8}(t-5)\right)$, has a period of $T = 16$. The function oscillates about $y = 7$ (vertical shift) with an amplitude of 4. The phase shift is $\phi = 5$. There is an absolute maximum at $(t_{max}, y(t_{max})) = (13, 11)$. There is an absolute minimum at $(t_{min}, y(t_{min})) = (5, 3)$. The graph of the function is below.



Problem 14

b. The equivalent form has $A = 7$, $B = 4$, and $\omega = \frac{\pi}{8}$. Since the amplitude has a negative sign, we phase shift the function by half a period. It follows that $\phi = 5 + 8 = 13$. Thus,

$$y(t) = 7 + 4\cos\left(\tfrac{\pi}{8}(t - 13)\right).$$

c. The notes show that the equivalent sine model is shifted a quarter period from the cosine model in Part b. Thus, $C = A$, $D = B$, $\nu = \omega$, and $\psi = \phi - 4$ or

$$y(t) = 7 + 4\sin\left(\tfrac{\pi}{8}(t - 9)\right).$$

15. a. Sum of squares of the errors is $J = 207.9$. The beagle is furthest from the model.

b. The length of a Borzoi is predicted to be $L(81) = 148$ cm. The model suggests that the Border Collie will have a height of 44 cm.

16. a. The square errors are

$$e_1^2 = 169 \qquad e_2^2 = 441 \qquad e_3^2 = 64.$$

The sum of squares of the errors is 674.

b. The square errors are

$$e_1^2 = 10.129 \qquad e_2^2 = 4.397 \qquad e_3^2 = 0.661.$$

The sum of squares of the errors is 15.19.

c. For the glucose model, there is no t-intercept. The G-intercept is $(0, 260)$. There are no vertical asymptotes. The horizontal asymptote is $G = 80$. The graph is below.

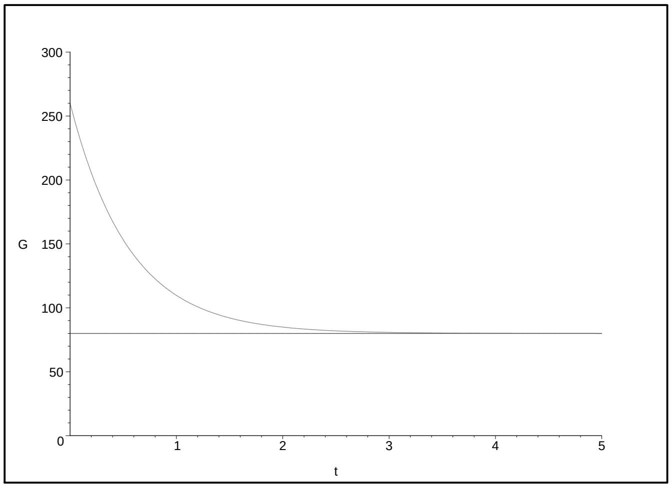


Figure 1: Glucose model (16.c)

17. a. The sum of squares of error for the model $A = mc$ and data is given by $J(m) = 46m^2 - 49.6m + 13.38$. The m-value of the vertex is given by $m = 0.5391$, and the least sum of squares function satisfies $J(0.5391) = 0.011$.

b. Using the best slope in the model, $A = 0.5391c$ gives the concentration of the unknown urea sample as $c = 4.08$ mM.

18. a. The sum of squares error satisfies for the model $d = kp$ and the data is given by $J(k) = 57.21k^2 - 68.72k + 20.65$. The k-value of the vertex is given by $k = 0.6006$, and the least sum of squares function satisfies $J(0.6006) = 0.0136$.

b. Using the best slope in the model, $d = 0.6006p$, the leopard shark would measure $d = 1.32$ m. A 2 m shark would appear on the photograph as $p = 3.3$ cm.

19. a. The linear model is given by $F = 0.4184W + 155.7$ with constants $m = 0.4184$ and $b = 155.7$.

b. The allometric model is given by $F = 3.329W^{0.7528}$ with constants $k = 3.329$ and $a = 0.7528$.

c. For a 1000 g chicken, the linear model says that the chicken consumes $F(1000) = 574.1$ g of feed per week. The allometric model says that this chicken consumes $F(1000) = 603.4$ g of feed per week.

If a chicken consumes 500 g of food, then the linear model suggests that the chicken weighs 822.9 g, while the allometric model suggests that the chicken weighs 779.0 g. The allometric model is superior as the linear model would suggest that a non-existent chicken ($W = 0$ g) would still eat 155.7 g of feed.

20. a. The linear model is $T = 5v + 1.75$ with constants $m = 5$ and $b = 1.75$. A 2 mm tissue sample gives a voltage drop of $v = 0.05$ V. If the experiment reveals a 0.6 V drop, then the tissue thickness is $T(0.6) = 4.75$ mm.

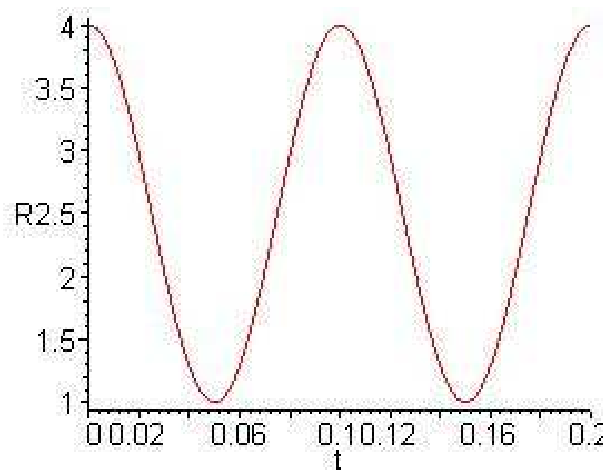
b. The allometric model is $T = 5.913V^{0.4894}$ with constants $k = 5.913$ and $a = 0.4894$. If the thickness is 2 mm, then the voltage drop is $v = 0.1092$V. If the voltage drop is 0.6V, then the thickness of the unknown tissue is $T(0.6) = 4.605$ mm.

c. The allometric model is superior because the voltage drop goes to zero when the thickness goes to zero.

21. a. The periodic contractions of 10/min implies that the period is 0.1 min. Thus, $0.1\omega = 2\pi$ or $\omega = 20\pi$. The average value $A = \frac{4+1}{2} = 2.5$, while the amplitude is given by $B = 4 - 2.5 = 1.5$. Thus, the radius of the small intestine is given by

$$R(t) = 2.5 + 1.5\cos(20\pi t).$$

b. The graph of $R(t)$ for $t \in [0, 0.2]$ is shown below. The maxima occur at $t = 0, 0.1, 0.2$ min, and the minima are halfway between the maxima with $t = 0.05, 0.15$ min.



c. The equivalent sine form of the model is phase shifted by a quarter period or $\frac{1}{40}$ min, so $\phi = 0 - \frac{1}{40}$. However, this is negative, so the principle phase shift requires adding one period or $\phi = -\frac{1}{40} + \frac{1}{10} = \frac{3}{40}$. The equivalent sine model is written:

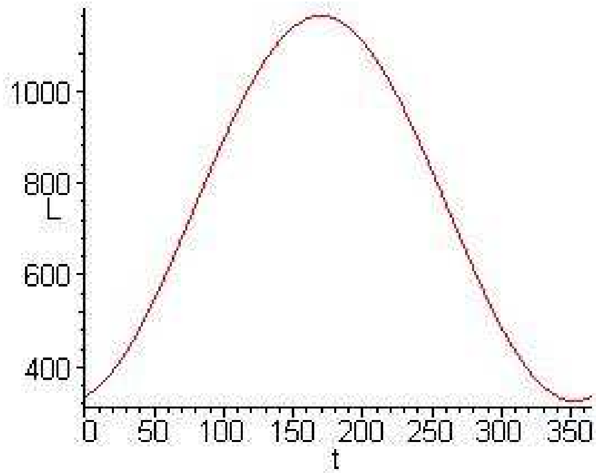
$$R(t) = 2.5 + 1.5\sin\left(20\pi\left(t - \frac{3}{40}\right)\right).$$

22. a. The period is 365 days, so $365\omega = 2\pi$ or $\omega = \frac{2\pi}{365} \simeq 0.01721$. The average length of time is $\alpha = \frac{1162+327}{2} = 744.5$ min. The amplitude is given by $\beta = 1162 - 744.5 = 417.5$ min. The

maximum occurs on day 170, so $\omega(170-\phi) = \pi/2$ (based on the maximum of the sine function). Thus, $170 - \phi = \frac{365}{4} = 91.25$ or $\phi = 78.75$ day. It follows that

$$L(t) = 744.5 + 417.5\sin(0.01721(t - 78.75)).$$

The length of day for Ground Hog's day is $L(32) = 744.5 + 417.5\sin(0.01721(32 - 78.75)) = 443.7$ min in Anchorage.



b. The equivalent cosine form of the model is phase shifted by a quarter period or 91.25 days, so $\phi = 78.75 + 91.25 = 170$. Also, one can use that the maximum of the cosine model occurs at the phase shift. Thus, the equivalent cosine model is written:

$$L(t) = 744.5 + 417.5\cos(0.01721(t - 170)).$$

23. a. From the high and low temperatures, A is the average, so $A = 18°$C. The amplitude B is the difference between the maximum and the average, so $B = 8°$C. The period is 24 hr, so $24\omega = 2\pi$ or $\omega = \frac{\pi}{12} \simeq 0.2618$. The maximum temperature occurs at 4 PM ($t = 16$), so

$$T(16) = 26 = 18 + 8\sin\left(\frac{\pi}{12}(16 - \phi)\right).$$

It follows that

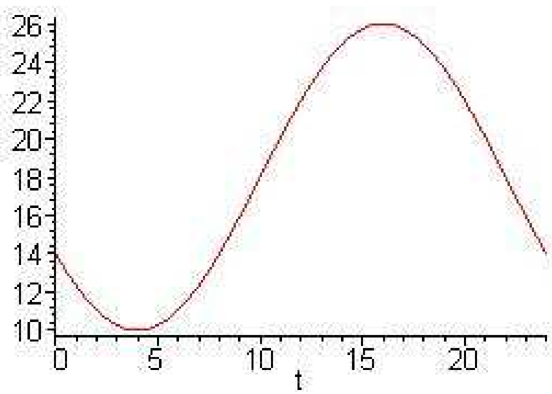
$$\sin\left(\frac{\pi}{12}(16 - \phi)\right) = 1 \quad \text{or} \quad \frac{\pi}{12}(16 - \phi) = \frac{\pi}{2}.$$

Hence, $\phi = 10$. The sine model becomes

$$R(t) = 18 + 8\sin(0.2618(t - 10)).$$

b. The equivalent cosine form of the model is phase shifted by a quarter period or 6 hr, so $\phi = 10 + 6 = 16$. Again the phase shift for the cosine model is easy as it corresponds to the maximum. So, we obtain the equivalent cosine model:

$$R(t) = 18 + 8\cos(0.2618(t - 16)).$$



24. a. $H_1 = 2040$ and $H_2 = 2080.8$. In the general solution is given by $H_n = (1.02)^n H_0 = 2000(1.02)^n$.

b. The general solution is given by $G_n = (1.03)^n G_0 = 200(1.03)^n$. It takes 23.5 generations for the population to double.

c. The populations are equal in 236 generations.

25. a. The annual growth rate is $r = 0.011753$ (about 1.2% per year) and the general equation is $P_n = (1.011753)^n 179.3$.

b. The model predicts a population of 286.1 million in the year 2000. The error between this and the actual population is 1.7%.

c. The population will double about 59.3 years after 1960, or about the year 2019.

26. a. For France, the growth rate is $r = 0.051948$ (about 5.2% per decade) and the general equation is $P_n = (1.051948)^n 53.9$.

b. The population predictions are 59.6 million in the year 2000 and 66 million in the year 2020. The error in the year 2000 is 0.34%.

c. In Kenya, the growth rate is $r = 0.4491$ (about 44.9% per decade) and the general equation is $P_n = (1.4491)^n 16.7$. The population predictions are 35.1 million in the year 2000 and 73.6 million in the year 2020. It takes 1.87 decades, or about 18.7 years, for the population of Kenya to double.

d. The populations become equal in 3.66 decades, so the population of Kenya will first exceed that of France in 2017. Population for France in 2017 is 65.0 million, while the population of Kenya in 2017 is 65.9 million.

e. It follows that the annual growth rate in France is $r = 0.00508$, while in Kenya it is $r = 0.0378$.

27. a. For Poland, the annual growth rate is $r = 0.01362$, and the population doubles in 51.25 years.

b. The model population is 48.81 million in the year 2000, which gives an error of 26.29%.

c. The equilibria for this logistic growth model are $P_{1e} = 0$ and $P_{2e} = 42.0$ million.

d. The populations is growing fastest when $P_v = 21.0$ million with a growth of $G(P_v) = 0.441$ million/yr.

28. a. The 3 populations are $p_1 = 700$, $p_2 = 860$, and $p_3 = 988$.

b. The equilibrium is $p_e = 1500$. The equilibrium is stable.

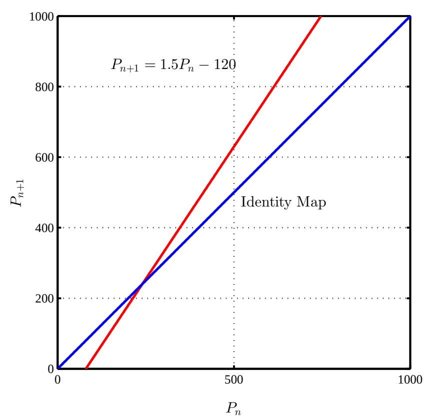
29. a. The breathing fraction is $q = 0.120536$, and the functional reserve capacity is $V_r = 2188.9$ ml.

b. The concentration of Helium in the next two breaths are $c_2 = 39.85$ and $c_3 = 35.67$. The equilibrium concentration is $c_e = \gamma = 5.2$ ppm of He, which is a stable equilibrium.

30. a. For the Malthusian growth model with dispersion, $P_{n+1} = (1 + r)P_n - \mu$, $r = 0.5$ and $\mu = 120$. The populations in the next two weeks ar $P_3 = 1117.5$ and $P_4 = 1556.25$.

b. The equilibrium is $P_e = 240$, and it is unstable.

c. The graph of the updating function and identity map, $P_{n+1} = P_n$, are shown below. The only point of intersection occurs at the equilibrium found above.



31. a. From the breathing model, $c_{n+1} = (1 - q)c_n + q\gamma$ and the data $c_0 = 400$, $c_1 = 352$, and $c_2 = 310$, we find the constants q and γ by substitution and the simultaneous solution of two equations and two unknowns. We have

$$352 = 400(1 - q) + q\gamma \qquad \text{and} \qquad 310 = 352(1 - q) + q\gamma.$$

Subtracting the second equation from the first gives $42 = 48(1 - q)$ or $1 - q = \frac{42}{48} = \frac{7}{8}$. Thus, $q = \frac{1}{8}$. This value is substituted into the first equation above to give $352 = 400\frac{7}{8} + \frac{1}{8}\gamma$, which gives $\gamma = 16$.

Thus, the model becomes $c_{n+1} = \frac{7}{8}c_n + 2$, and the next 2 breaths satisfy

$$\begin{aligned} c_3 &= \frac{7}{8}(310) + 2 = 273.25 \\ c_4 &= \frac{7}{8}(273.25) + 2 = 241.1 \end{aligned}$$

b. At the equilibria, $c_e = \frac{7}{8}c_e + 2$, so $\frac{1}{8}c_e = 2$ or $c_e = 16$, which is the value of γ as expected. This equilibrium is stable.

c. The graph of the updating function and identity map, $c_{n+1} = c_n$, are shown below. The only point of intersection occurs at the equilibrium, γ found above.