Computer Problem

1. a. Consider a one-dimensional rod that is insulated along its edges. Assume that it has a length of 10 cm. The rod is initially placed so that one end is 0°C and the other end is 100°C. It is allowed to come to a steady-state temperature distribution. Find this temperature distribution, $u_e(x)$.

b. At time $t = 0$, the one-dimensional rod from Part a is insulated on both ends. This implies that the rod satisfies the PDE:

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} &= \frac{\partial^2 u(x,t)}{\partial x^2}, \qquad t > 0, \quad 0 < x < 10, \\ \text{Boundary Conditions :} \quad & \frac{\partial u(0,t)}{\partial x} = 0, \qquad \frac{\partial u(L,t)}{\partial x} = 0, \quad t > 0, \\ \text{Initial Conditions :} \quad & u(x,0) = u_e(x), \quad 0 < x < L, \end{aligned}$$

where $u_e(x)$ is the steady state temperature distribution from Part a. Find the solution to this problem, including the Fourier coefficients. Create a graphic simulation showing the 3D plot of temperature as a function of t and x, using 20 and 200 terms (Fourier coefficients) to approximate the solution with $t \in [0, 20]$.