I, ________________________________(your name), pledge that this exam is completely my own work, and that I did not take, borrow or steal work from any other person, and that I did not allow any other person to use, have, borrow or steal portions of my work. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

1. a. The displacement of a uniform thin beam in a medium that resists motion satisfies the beam equation:

$$\frac{\partial^4 u}{\partial x^4} = -\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - k\frac{\partial u}{\partial t}, \quad 0 < x < a, \quad t > 0,$$

where $k < \frac{2\pi^2}{a^2 c}$. If the beam is simply supported at the ends, then the boundary conditions are:

$$u(0,t) = 0, \quad u_{xx}(0,t) = 0, \quad u(a,t) = 0, \quad u_{xx}(a,t) = 0.$$

Assume that there is initially no displacement and that an initial velocity, $u_t(x,0) = 1$ is given to the beam. Solve this initial-boundary value problem. You can assume that the eigenvalues are real, but show clearly how you obtain all eigenvalues and eigenfunctions.

b. Let $a = 2$, $c = 1$, and $k = 0.1$. Use 20 terms in the series solution of $u(x,t)$ and have the computer graph the displacement of the beam at times $t = 0$, 1, 2, 5, 10, and 20.

2. If convection is taken into account, the equation for heat conduction and convection in a one-dimensional rod is given by:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - v_0\frac{\partial u}{\partial x}, \quad 0 < x < L, \quad t > 0.$$

Let $k = 1$, $v_0 = 0.5$, and $L = 10$. Assume the following boundary conditions and initial conditions:

$$u(0,t) = 0, \quad u(L,t) = 0, \quad \text{and} \quad u(x,0) = f(x).$$

a. Use separation of variables to create two ordinary differential equations.

b. From the spatial ordinary differential equation, create a Sturm-Liouville eigenvalue problem. Identify explicitly the functions $p(x)$, $q(x)$, and $\sigma(x)$. Find the eigenvalues and eigenfunctions for this problem.

c. Solve the original partial differential equation with its boundary and initial conditions. Write clearly your integral for finding the Fourier coefficients.

3. Find the eigenvalues and eigenfunctions to the following:

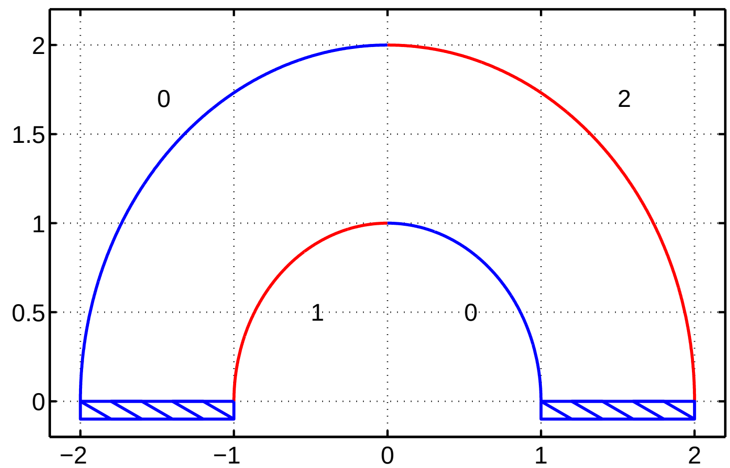
a. $u'' + (\lambda + 1)u = 0, \quad u(0) = 0, \quad u'(4) = 0.$

b. $u'' + 2\,u' + (1 - \lambda)u = 0, \quad u(0) = 0, \quad u(1) = 0.$

State clearly the orthogonality relationship for each of the above problems.

4. Find the steady-state temperature distribution for the Figure below (assuming the faces are insulated). The region is a semi-annular region satisfying Laplace's equation, where the edges along the x-axis are insulated. Along the semi-circular edges, we have:

$$u(1,\theta) = \begin{cases} 0, & 0 \le \theta < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} \le \theta \le \pi \end{cases}, \qquad u(2,\theta) = \begin{cases} 2, & 0 \le \theta < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le \theta \le \pi \end{cases}.$$



5. Consider the heat equation given by:

$$\frac{\partial u}{\partial t} = k\nabla^2 u, \quad 0 < x < L, \quad 0 < y < H, \quad t > 0.$$

With boundary conditions and initial condition:

$$\frac{\partial u}{\partial x}(0,y,t) = Ay, \quad \frac{\partial u}{\partial x}(L,y,t) = y^2, \quad \frac{\partial u}{\partial y}(x,0,t) = 0, \quad \frac{\partial u}{\partial y}(x,H,t) = 0, \quad \text{and} \quad u(x,y,0) = xy.$$

Find the condition on A that allows the steady state problem to be solvable on the rectangular domain. Solve the steady state problem.