1. Let $y_{n+1} = 0.7y_n + 6$ with $y_0 = 10$. Then

$$\begin{aligned}
y_1 &= 0.7y_0 + 6 &= 0.7(10) + 6 &= 13 \\
y_2 &= 0.7y_1 + 6 &= 0.7(13) + 6 &= 15.1 \\
y_3 &= 0.7y_2 + 6 &= 0.7(15.1) + 6 &= 16.57
\end{aligned}$$

Equilibrium occurs when $y_{n+1} = y_n = y_e$, so $y_e = 0.7y_e + 6$ or $0.3y_e = 6$. Thus, $y_e = 20$. The solution is approaching the equilibrium, so it is stable.

2. Let $z_{n+1} = 1.2z_n - 20$ with $z_0 = 50$. Then

$$\begin{aligned}
z_1 &= 1.2z_0 - 20 &= 1.2(50) - 20 &= 40 \\
z_2 &= 1.2z_1 - 20 &= 1.2(40) - 20 &= 28 \\
z_3 &= 1.2z_2 - 20 &= 1.2(28) - 20 &= 13.6
\end{aligned}$$

Equilibrium occurs when $z_{n+1} = z_n = z_e$, so $z_e = 1.2z_e - 20$ or $0.2z_e = 20$. Thus, $z_e = 100$. The solution is moving away from the equilibrium, so it is unstable.

3. This breathing model is given by $c_{n+1} = 0.9c_n + 0.1(5.2)$, with $c_0 = 100$. Then

$$\begin{aligned}
c_1 &= 0.9c_0 + 0.52 = 0.9(100) + 0.52 = 90.52\text{ppm} \\
c_2 &= 0.9c_1 + 0.52 = 0.9(90.52) + 0.52 = 81.99\text{ppm} \\
c_3 &= 0.9c_2 + 0.52 = 0.9(81.99) + 0.52 = 74.31\text{ppm}
\end{aligned}$$

Equilibrium occurs when $c_{n+1} = c_n = c_e$, so $c_e = 0.9c_e + 0.52$ or $0.1c_e = 0.52\text{ppm}$. Thus, $c_e = 5.2\text{ppm}$. The solution is approaching the equilibrium, so it is stable.

4. From the mathematical model $c_{n+1} = (1-q)c_n + q\gamma$ with $c_0 = 0.68$ and $c_1 = 0.694$, it follows that

$$0.694 = (1 - q)0.68 + 0.78q.$$

Therefore, it follows that

$$(0.78 - 0.68)q = 0.694 - 0.68 \qquad \text{or} \qquad q = 0.14.$$

The functional reserve capacity, given that $V_i = 400$ ml, satisfies

$$V_r = \frac{(1-q)V_i}{q} = \frac{0.86(400)}{0.14} = 2457 \text{ ml}.$$

Thus, the values $c_{n+1} = (1 - 0.14)c_n + 0.14(.78)$ with $c_0 = 0.68$. Then

$$\begin{aligned}
c_1 &= 0.86c_0 + 0.1092 = 0.86(0.68) + 0.1092 = 0.694 \\
c_2 &= 0.86c_1 + 0.1092 = 0.86(0.694) + 0.1092 = 0.706 \\
c_3 &= 0.86c_2 + 0.1092 = 0.86(0.706) + 0.1092 = 0.716
\end{aligned}$$

Equilibrium occurs when $c_{n+1} = c_n = c_e$, so $c_e = 0.86c_e + 0.1092$ or $0.14c_e = 0.1092$. Thus, $c_e = 0.78$. The solution is approaching the equilibrium, so it is stable.

5. Let $P_{n+1} = 1.05P_n + 200$, with $P_0 = 1000$. Then

$$
\begin{aligned}
P_1 &= 1.05P_0 + 200 = 1.05(1000) + 200 = 1250 \\
P_2 &= 1.05P_1 + 200 = 1.05(1250) + 200 = 1512.5 \simeq 1513 \\
P_3 &= 1.05P_2 + 200 = 1.05(1513) + 200 \simeq 1788
\end{aligned}
$$

6. a. The model gives the following

$$
\begin{aligned}
P_{n+1} &= (1+r)P_n - \mu \\
90 &= 70(1+r) - \mu \\
150 &= 100(1+r) - \mu \\
250 &= 150(1+r) - \mu
\end{aligned}
$$

Subtracting we have $150 - 90 = (100 - 70)(1 + r)$ so

$$
1 + r = \frac{60}{30} = 2 \qquad \text{or} \qquad r = 1.
$$

But $150 = 100(1 + 1) - \mu$, so $\mu = 50$.

b. To find the populations P_1, P_2, and P_3,

$$
\begin{aligned}
P_{n+1} &= 2P_n - 50 \\
P_1 &= 2(100) - 50 = 150 \\
P_2 &= 2(150) - 50 = 250 \\
P_3 &= 2(250) - 50 = 450
\end{aligned}
$$

c. Equilibrium occurs when $P_{n+1} = P_n = P_e$, so $P_e = 2P_e - 50$ or $P_e = 50$. The solution is moving away from the equilibrium, so it is unstable.

7. a. The population of a species of moth satisfies the model

$$
P_{n+1} = rP_n + \mu.
$$

From the data in 1990, 1991, and 1992 with populations of $P_0 = 6000$, $P_1 = 5500$, and $P_2 = 5100$, respectively, the model gives the following:

$$
\begin{aligned}
5500 &= 6000r + \mu \\
5100 &= 5500r + \mu.
\end{aligned}
$$

By subtracting these equations, we have $5500 - 5100 = (6000 - 5500)r$, so $r = \frac{400}{500} = 0.8$. But $5500 = 6000(0.8) + \mu$, so $\mu = 700$. Therefore, we can find the number of moths in the following 3 years (1993, 1994, and 1995) with the following computation:

$$
\begin{aligned}
P_3 &= 0.8P_2 + 700 = 0.8(5100) + 700 = 4780 \\
P_4 &= 0.8P_3 + 700 = 0.8(4780) + 700 = 4524 \\
P_5 &= 0.8P_4 + 700 = 0.8(4524) + 700 = 4319
\end{aligned}
$$

b. As before, the equilibrium satisfies $P_{n+1} = P_n = P_e$, so $P_e = 0.8P_e + 700$ or $0.2P_e = 700$, which gives $P_e = 3500$. This population is moving towards the equilibrium, so it is considered stable. The equilibrium population is the long time behavior of the model in this case, so ultimately, we expect 3500 moths on the island.

c. The graph of the updating function and the identity map are below. Note that the equilibrium population is where the updating function and identity map intersect. Also included is the cobwebbing for the first few iterations, showing the solution heading toward the equilibrium.

Moth population