# Calculus for the Life Sciences I

## Lecture Notes – Quadratic Equations and Functions

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# Outline

# Weak Acids

- Many of the organic acids found in biological applications are weak acids

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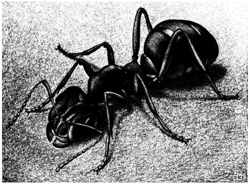
- Many of the organic acids found in biological applications are weak acids
- Weak acid chemistry is important in preparing buffer solutions for laboratory cultures

# Formic Acid



**Ants**

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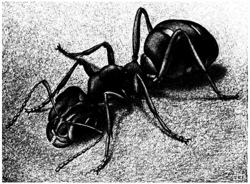

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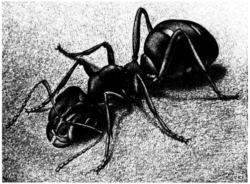

## Ants

- Formic acid (HCOOH) is a relatively strong weak acid that ants use as a defense
- The strength of this acid makes the ants very unpalatable to predators

# Acid Chemistry

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$$HCOOH \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} H^+ + HCOO^-.$$

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- For formic acid, $K_a = 1.77 \times 10^{-4}$
- Let $[X]$ denote the concentration of chemical species $X$
- Formic acid is in equilibrium, when:

$$K_a = \frac{[H^+][HCOO^-]}{[HCOOH]}$$

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- It follows that $[HCOOH] = x - [H^+]$
- Thus,

$$K_a = \frac{[H^+][H^+]}{x - [H^+]}$$

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The previous equation is written

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Only the positive solution is taken to make physical sense

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The equation above gives

$$[H^+] = \frac{1}{2}\left(-0.000177 + \sqrt{(0.000177)^2 + 4(0.000177)(0.1)}\right)$$

or

$$[H^+] = 0.00412$$

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Since pH is defined to be $-\log_{10}[H^+]$, this solution has a pH of 2.385

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Three methods for solving quadratics:

1. Factoring the equation
2. The **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. Completing the Square

# Example of Factoring a Quadratic Equation

**Consider the quadratic equation:**

$$x^2 + x - 6 = 0$$

Find the values of $x$ that satisfy this equation.

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Thus,

$$x = -3 \quad \text{and} \quad x = 2$$

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or

$$x = -2.732 \quad \text{and} \quad x = 0.732$$

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This has no real solution, only the complex solution

$$x = 2 \pm i$$

# Quadratic Function

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$$f(x) = ax^2 + bx + c,$$

**where $a \neq 0$ and $b$ and $c$ are arbitrary.**

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The graph of

$$y = f(x)$$

produces a parabola

# Vertex

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The **Vertex of the Parabola** is the point

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The parameter $a$ determines the direction the parabola opens

- If $a > 0$, then the parabola opens upward
- If $a < 0$, then the parabola opens downward
- As $|a|$ increases the parabola narrows

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Consider the functions

$$f_1(x) = 3 - 2x \quad \text{and} \quad f_2 = x^2 - x - 9$$

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- Find the slope of the line
- Find the vertex of the parabola
- Find the points of intersection
- Graph the two functions

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- Has $y$-intercept $y = 3$
- Has $x$-intercept $x = \frac{3}{2}$
- Has slope $m = -2$

# Example of Line and Parabola 3

**Solution (cont):** The parabola

$$f_2 = x^2 - x - 9$$

- Has $y$-intercept $y = -9$, since $f_2(0) = -9$

# Example of Line and Parabola 3

**Solution (cont):** The parabola

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- Has $y$-intercept $y = -9$, since $f_2(0) = -9$
- By quadratic formula the $x$-intercepts satisfy

$$x = \frac{1 \pm \sqrt{37}}{2} \quad \text{or} \quad x \approx -2.541, 3.541$$

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- Vertex satisfies $x = \frac{1}{2}$ and $y = -\frac{37}{4}$

## Example of Line and Parabola 4

**Solution (cont):** The points of intersection of

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$$3 - 2x = x^2 - x - 9 \quad \text{or} \quad x^2 + x - 12 = 0$$

# Example of Line and Parabola



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Factoring

$$(x+4)(x-3) = 0 \quad \text{or} \quad x = -4, 3$$

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Factoring

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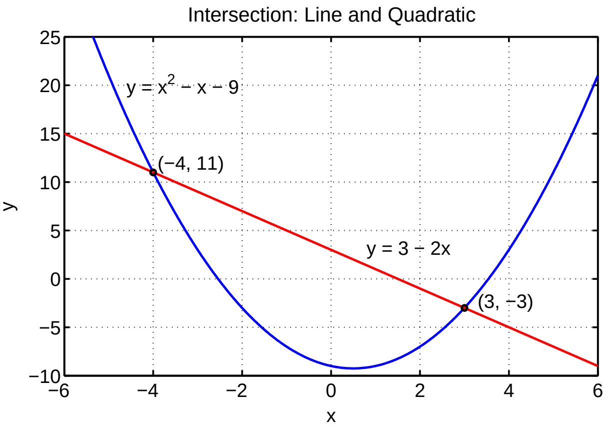
Points of intersection are

$$(x_1, y_1) = (-4, 11) \quad \text{or} \quad (x_2, y_2) = (3, -3)$$

# Example of Line and Parabola



**Solution (cont):** Graph of the functions



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- Sketch a graph of $h(t)$ vs. $t$
- Find the maximum height of the ball
- Determine when the ball hits the ground

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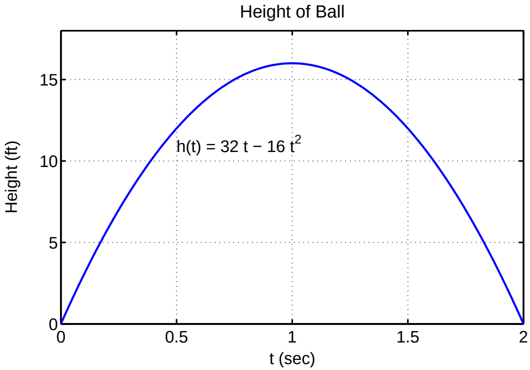
The midpoint between the intercepts is $t = 1$

Thus, the vertex is $t_v = 1$, and $h(1) = 16$

# Example of Line and Parabola 3

**Solution (cont):** The graph is



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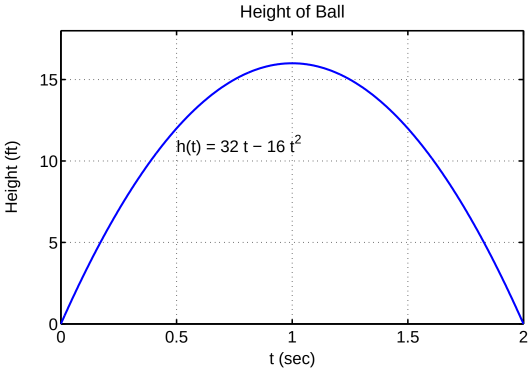


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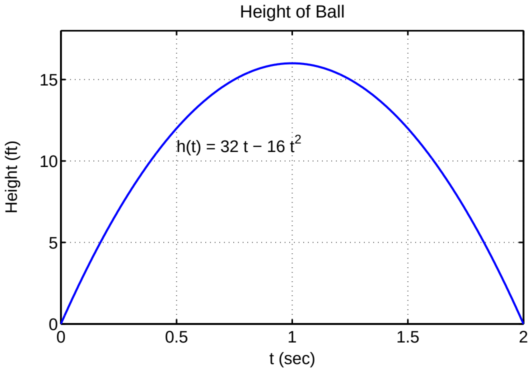


- The maximum height of the ball is 16 ft
- The ball hits the ground at $t = 2$ sec

# Lambert-Beer Law



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## Concentration and Absorbance

- A spectrophotometer uses the Lambert-Beer law to determine the concentration of a sample ($c$) based on the absorbance of the sample ($A$)
- The ion dichromate forms an orange/yellow that has a maximum absorbance at 350 nm and is often used in oxidation/reduction reactions
- The **Lambert-Beer law** for the concentration of a sample from the absorbance satisfies the linear model

$$c = mA$$

where $m$ is the slope of the line (assuming the spectrophotmeter is initially zeroed)

# Lambert-Beer Law



## Spectrophotometer data for an redox reaction

- Data collected on some known samples

| $A$ | 0.12 | 0.32 | 0.50 | 0.665 |
|---|---|---|---|---|
| $c$ (mM) | 0.05 | 0.14 | 0.21 | 0.30 |

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- Sketch a graph of $J(m)$ and find the vertex of this quadratic function
- Sketch a graph of the data and the line that best fits the data
- Use this model to determine the concentration of two unknown samples that have absorbances of $A = 0.45$ and 0.62

# Lambert-Beer Law



**Solution:** Given the linear model $c = mA$, the sum of square errors satisfies

$$
\begin{aligned}
J(m) &= e_1^2 + e_2^2 + e_3^2 + e_4^2 \\
&= (0.05 - 0.12m)^2 + (0.14 - 0.32m)^2 + (0.21 - 0.50m)^2 + (0.30 - 0.66m)^2 \\
&= 0.8024m^2 - 0.7076m + 0.1562
\end{aligned}
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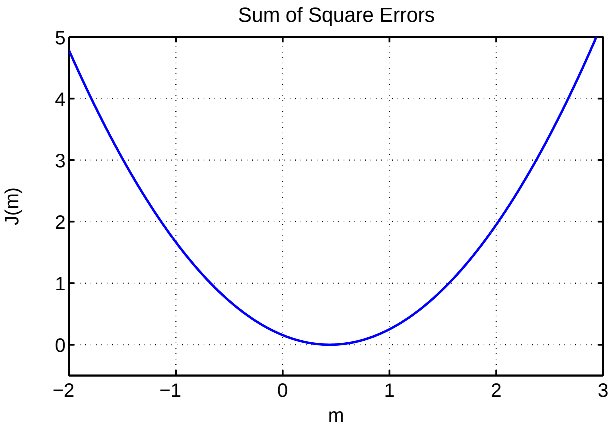
The vertex has $m_v = \frac{0.7076}{2(0.8024)}$, so

$$(m_v, J(m_v)) = (0.44093, 0.00019995)$$

# Lambert-Beer Law



**Solution (cont):** Graph of $J(m)$



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**Solution (cont):** Since the vertex has $m_v = 0.44093$, the **best linear model** is

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For an absorbance $A = 0.45$

$$c(0.45) = 0.441(0.45) = 0.198$$

# Lambert-Beer Law



**Solution (cont):** Since the vertex has $m_v = 0.44093$, the **best linear model** is

$$c = 0.441\, m$$

For an absorbance $A = 0.45$

$$c(0.45) = 0.441(0.45) = 0.198$$

The best model predicts a concentration of 0.198 nM

## Lambert-Beer Law



**Solution (cont):** Since the vertex has $m_v = 0.44093$, the **best linear model** is

$$c = 0.441\, m$$

For an absorbance $A = 0.45$

$$c(0.45) = 0.441(0.45) = 0.198$$

The best model predicts a concentration of 0.198 nM

For an absorbance $A = 0.62$

$$c(0.62) = 0.441(0.62) = 0.273$$

# Lambert-Beer Law



**Solution (cont):** Since the vertex has $m_v = 0.44093$, the **best linear model** is

$$c = 0.441\, m$$

For an absorbance $A = 0.45$

$$c(0.45) = 0.441(0.45) = 0.198$$

The best model predicts a concentration of 0.198 nM

For an absorbance $A = 0.62$

$$c(0.62) = 0.441(0.62) = 0.273$$

The best model predicts a concentration of 0.273 nM

# Lambert-Beer Law



**Solution (cont):** Graph of Best Linear Model and Data