Calculus for the Life Sciences I

Lecture Notes – Other Functions and Asymptotes

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Outline

Enzyme Kinetics

Proteins

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- Enzymes are proteins that facilitate reactions inside the cell
- Enzymes are noted for their specificity and speed under a narrow range of conditions
  - β-galactosidase catalyzes the break down of lactose into glucose and galactose
  - Urease rapidly converts urea into ammonia and carbon dioxide

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- The law of mass action applied to biochemical equations
- Differential equations are formed (Math 122)
- Simplifications for basic reactions
  - The enzyme-substrate complex forms extremely rapidly, creating a **quasi-steady state**
  - The forward reaction or **turnover number**, k_2, occurs on a slower time scale

Enzyme Production Rate

The **Michaelis-Menten reaction rate** for product

$$R([\mathrm{S}]) = \frac{k_2[\mathrm{E}_0][\mathrm{S}]}{K_m + [\mathrm{S}]} = \frac{V_{max}[\mathrm{S}]}{K_m + [\mathrm{S}]},$$

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- V (or V_{max}) is called the **maximal velocity of the reaction**
- K_m is the **Michaelis constant**
- K_m is substrate concentration at which the reaction achieves half of the maximum velocity

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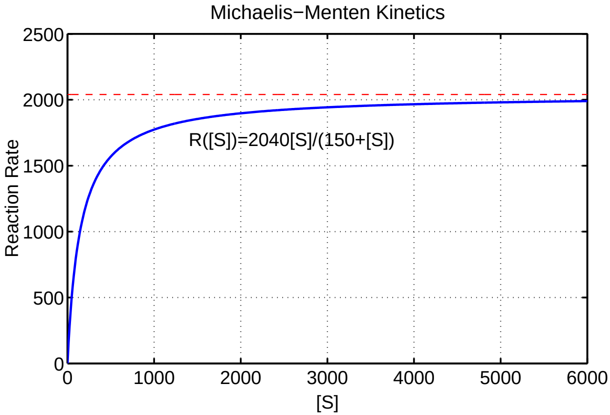
- As the concentration increases, there are diminishing returns with the eventual saturation of the reaction at some maximal rate

Binding of ATP to Myosin

Graph of the binding of ATP to Myosin



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- Quadratic functions are second order polynomials

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- These phenomena are topics that **Calculus** covers

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- Easy to use in approximations or numerical methods

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Consider the cubic polynomial given by

$$p(x) = x^3 - 3x^2 - 10x$$

Find the roots of this equation and graph this cubic polynomial

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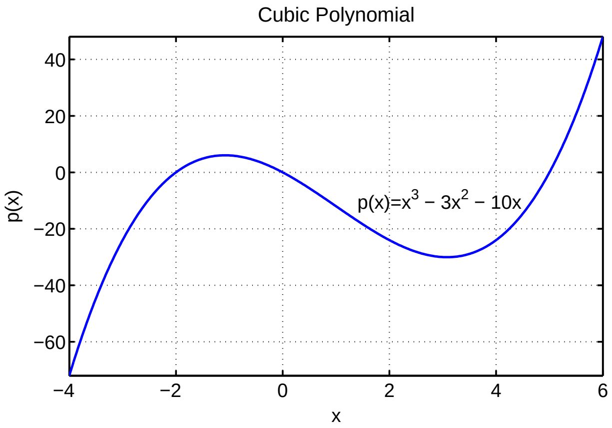
Example of Cubic Polynomial 2

Solution (cont): The graph is



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The roots of this polynomial are $x = -2, -1, 1, 2$

Rational Functions

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- The roots of the polynomial $q(x)$ are candidates for **vertical asymptotes** of $r(x)$
- When the order of the polynomial in the numerator of a rational function is less than or equal to the order of the polynomial of the denominator, then a **horizontal asymptote** occurs

Vertical Asymptote

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- A function cannot continuously cross a vertical asymptote
- Most of the time a rational function, $r(x) = \frac{p(x)}{q(x)}$ has a vertical asymptote at $x = a$ when $q(a) = 0$

Horizontal Asymptote

Definition: When the graph of a function $f(x)$ approaches a horizontal line, $y = c$, as x becomes very large and positive ($x \to \infty$), or x becomes very large and negative ($x \to -\infty$), then the line, $y = c$, is called a **horizontal asymptote**

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Note that a function can cross a horizontal asymptote for "small" values of x

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3. If $n = m$, then $r(x)$ has a horizontal asymptote of $y = a_n/b_n$.

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Consider the sequence of numbers

$$x_n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \frac{1}{k} \dots \qquad \text{for} \qquad n = 2, 3, 4, \dots, k, \dots$$

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These numbers are getting closer and closer to zero

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Since

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Thus, there is a **vertical asymptote** at $x = 0$

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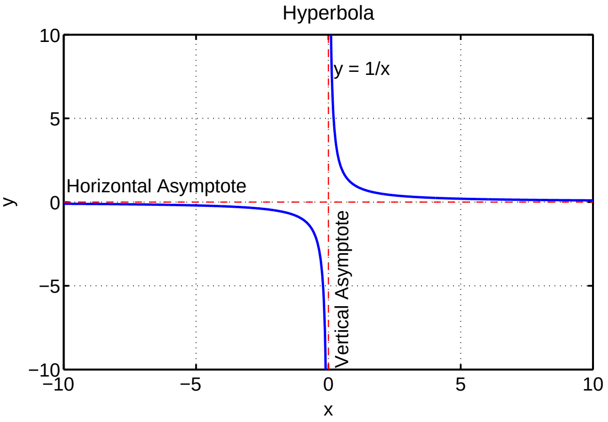
The graph of $y = \frac{1}{x}$ is



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2. Find the x and y-intercepts
3. Find vertical and horizontal asymptotes
4. Graph the function

Rational Function Example 1 2

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The edge of the domain is $x = -2$, so we see there is a **vertical asymptote** at $x = -2$

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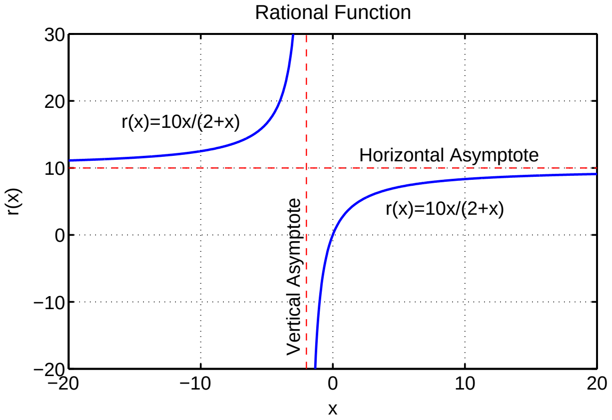
Thus, a **horizontal asymptote** occurs at $y = 10$

Rational Function Example 1

The graph of $y = \frac{10x}{2+x}$ is



Rational Function Example 2

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where $p(x) = x + 2$ and $q(x) = x - 3$

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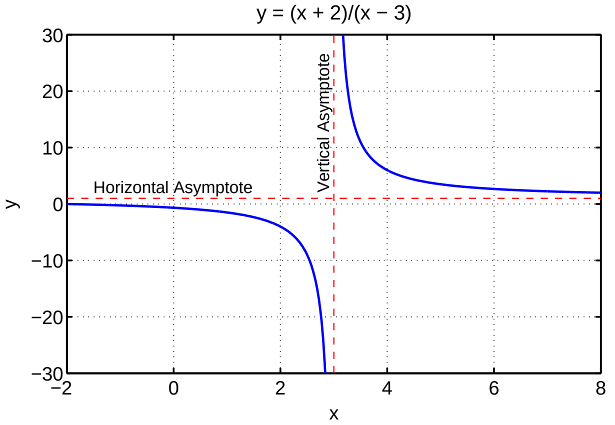
Rational Function Example 2

The graph of $y = \frac{x+2}{x-3}$ is



Rational Function Example 3

Consider the rational function

$$f(x) = \frac{4x^2}{4 - x^2}$$

where $p(x) = 4x^2$ and $q(x) = 4 - x^2$

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1. Find the domain of the function
2. Find the x and y-intercepts
3. Find vertical and horizontal asymptotes
4. Graph the function

Rational Function Example 3 2

Solution: The denominator is zero when $x = \pm 2$ so the domain is

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Note that this function is an **even function**

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Note that this function is an **even function**

The edge of the domain is $x = \pm 2$, so we see there are **vertical asymptotes** at $x = \pm 2$

Rational Function Example 3

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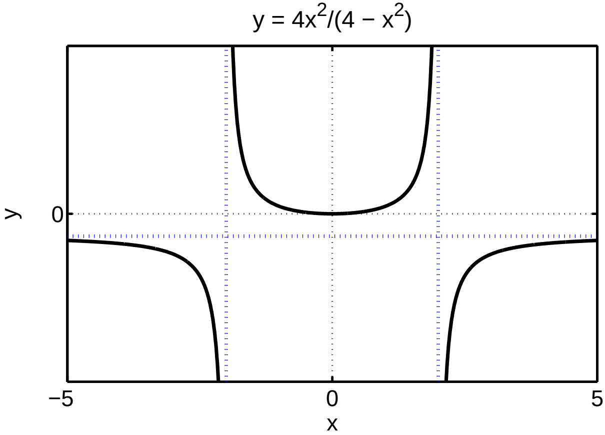

Lineweaver-Burk Plot 1

The **Michaelis-Menten rate function** traces out a hyperbola

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Lineweaver-Burk Plot 1

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The inverse of this expression is written

$$\frac{1}{V} = \frac{K_m + [\mathrm{S}]}{V_{max}[\mathrm{S}]} = \frac{K_m}{V_{max}}\frac{1}{[\mathrm{S}]} + \frac{1}{V_{max}}$$

Lineweaver-Burk Plot 2

The inverse expression is **linear** in $\frac{1}{[S]}$ and $\frac{1}{V}$

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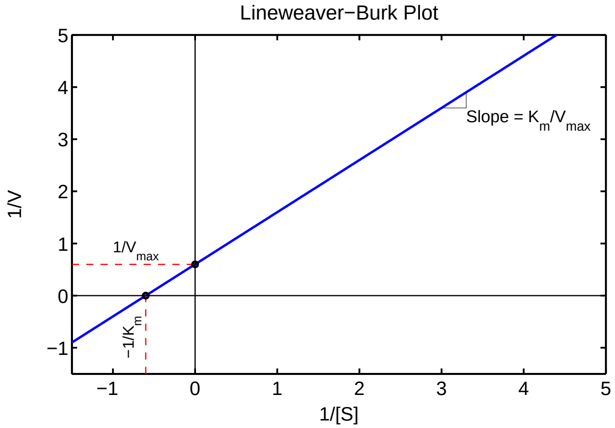
- The x-intercept is $-1/K_m$

Lineweaver-Burk Plot

Below is the Lineweaver-Burk Plot



Lineweaver-Burk Plot 4

The **Lineweaver-Burk Plot** provides a valuable method for experimentally measuring the characteristics of an enzyme

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If the data aren't linear, then the enzyme is not Michaelis-Menten type

Enzyme Example

Suppose an enzyme satisfies the equation

$$V = \frac{20[S]}{10 + [S]}$$

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- Find the enzyme's characteristic parameters, K_m and V_{max}

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Solution: The graph passes through the origin with no vertical asymptotes in the domain $[S] \geq 0$

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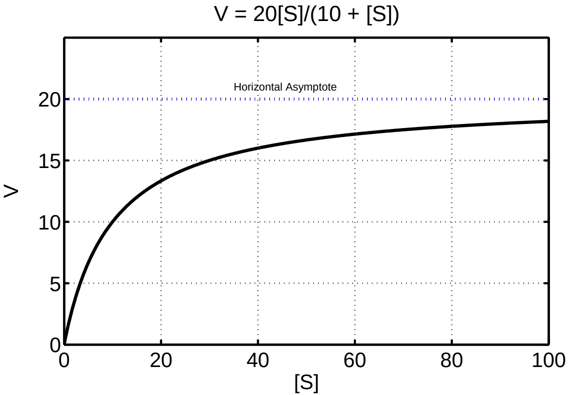
This gives a horizontal asymptote of $V = 20$

Enzyme Example

Graph of rational function for enzyme



Enzyme Example 4

Solution (cont): The Lineweaver-Burk formulation looks at the inverse of the enzyme reaction formula

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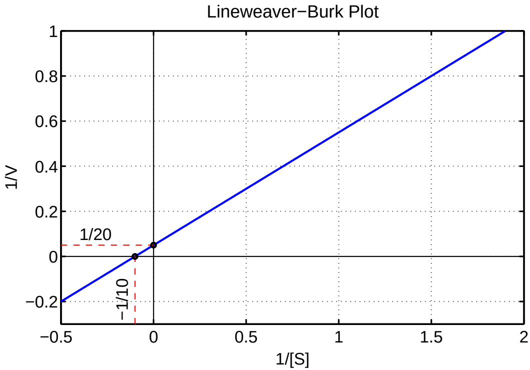
Since the y-intercept is $1/V_{max} = \frac{1}{20}$,
so $V_{max} = 20$

Enzyme Example

The slope is $K_m/V_{max} = \frac{1}{2}$, so $K_m = 10$



Weak Acid Chemistry

Weak Acid Chemistry

A weak acid with equilibrium constant, K_a, and normality, x, was shown to have acid concentration

$$[H^+] = \frac{1}{2}\left(-K_a + \sqrt{K_a^2 + 4K_a x}\right)$$

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$$[H^+] = \frac{1}{2}\left(-K_a + \sqrt{K_a^2 + 4K_a x}\right)$$

The $[H^+]$ is a **square root function** of the normality, x

Formic Acid - $[H^+]$ 1

Formic Acid has an equilibrium constant, $K_a = 1.77 \times 10^{-4}$

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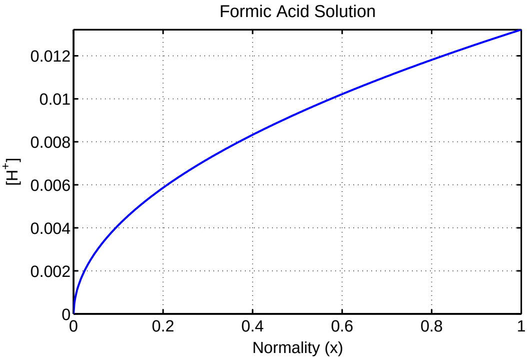
Below is a graph of $[H^+]$



Formic Acid - pH

The concentration of $[H^+]$ for **Formic Acid** was graphed above
The pH of the solution is $-\log_{10}([H^+])$

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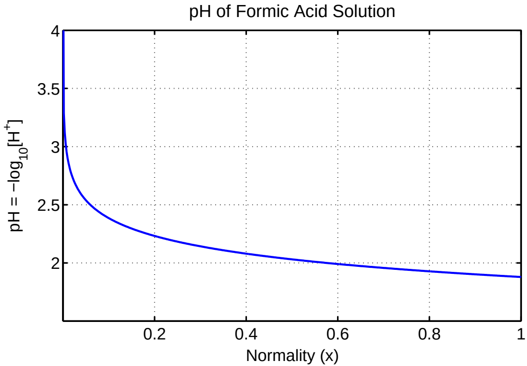
Below is a graph of the pH



Square Root Function

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- The square root function is only defined for positive quantities under the radical
- **The domain of a square root function is found by solving the inequality for the function under the radical being greater than zero**

Example of Square Root Function 1

Consider the function

$$y = \sqrt{x+2}$$

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Find the domain of this function and graph the function

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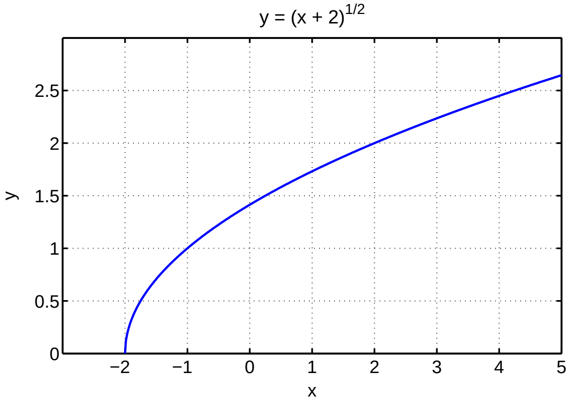
This example has its function defined for $x \geq -2$

Example of Square Root Function

Below is a graph of $y = \sqrt{x+2}$



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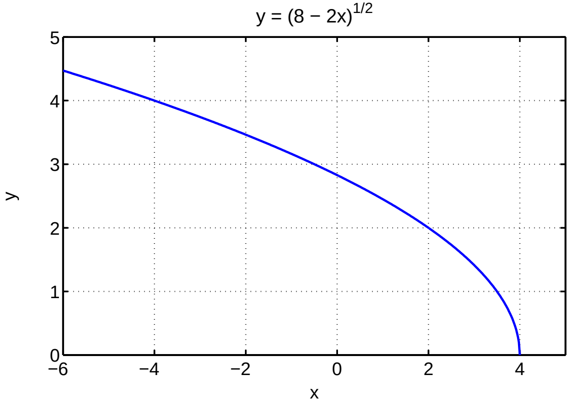
The range is all $y \geq 0$

Example2: Square Root Function 2

Below is a graph of $y = \sqrt{8 - 2x}$



Example 3: Square Root Function 1

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Since the maximum occurs at $x = -1$ (with $y(-1) = \sqrt{9}$)
The range is $0 \leq y \leq 3$

Example of Square Root Function

Below is a graph of $y = \sqrt{8 - 2x - x^2}$